Exponentionation

1. Exponentiation

– Exponention and logarithm are connected with following equations:

$$x = b^{n/d}$$

$$\log_b x = \frac{n}{d}$$

where:
- x – result of exponentitation,
- b – base,
- n – power's numerator,
- d – power's denominator.

– Following rules can be used to make calculations with exponentitation more simple:

$$x = b^{n/d} \;\Big/ \uparrow^{d} \quad \Rightarrow \quad x^d = b^n$$

$$x^0 = 1$$

$$x^{-1} = \frac{1}{n}$$

1.1. Examples

If **positive integer** is used for power exponentionation is interpreted as multiple multiplications:

$$10^3 = \underbrace{1 \cdot 10 \cdot 10 \cdot 10}_{3}$$

If **zero** is used for power, using the above definition, we can conclude that no multiplication should take place, which results in:

$$10^0 = 1$$

If **-1** is used for power, exponentionation is interpreted as dividing 1 by the base:

$$10^{-1} = \frac{1}{10}$$

Having in mind the above three rules we can define exponentiation generaly like this:

$$x = 10^{-5/8} \;\Big/ \uparrow^{8}$$
$$x^8 = 10^{-5}$$
$$x^8 = 10^{-1 \cdot 5} = \left(10^{-1}\right)^5$$
$$x^8 = \left(\frac{1}{10}\right)^5$$

Result of exponentiation is number which multiplied by itself by fraction's denominator number of times gives number equal to multipling base by itself by fraction's nominator number of times.

Alternative way of representing power which are fractions with 1 as nominator is to use n-th root:

$$1000^{1/3} = \sqrt[3]{1000}$$

If **negative integer** is used for power is used then exponentionation can be interpreted as multiple divisions:

$$10^{-3} = \underbrace{1:10:10:10}_{3}$$

Using **positive fraction** for exponent, in which nominator is 1, can be interpreted as finding number which multiplied by itself by
fractions denominator number of times, results in base:

$$1000^{1/3} = 10 \diagup \uparrow^{3}$$

$$1000 = 10^{3} = 1 \cdot 10 \cdot 10 \cdot 10$$

Using **negative fraction** for exponent, in which nominator is 1, can be interpreted as finding number which diveded by itself by
fractions denominator number of times, results in base

$$0.001^{-1/3} = 10 \diagup \uparrow^{-3}$$

$$0.001 = 10^{-3} = 1:10:10:10$$

Logarithm of some number tels us to which potention must we raise base of the logarithm to get that number:

$$\log_{10} 1000 = 3$$

$$10^{3} = 1000$$

- **Exponentionation with complex numbers**

We will start by showing how number 1 can be expressed in complex notation:

$$1 = e^{i2\pi n} = \cos 2\pi n + i\sin 2\pi n = 1 + i \cdot 0 = 1 \ , n \in Z$$

We shall now calculate number one rased to 1/4 in two different ways.
First we shall do this using above definition, which means finding number which multiplied by itself 4 times gives number 1:

$$1^{1/4} = x \ / \uparrow^4$$

$$1 = x^4 = 1 \cdot x \cdot x \cdot x \cdot x \Rightarrow x=1,-1,i,-1$$

The same result can be optained using folowing procedure that deals with complex numbers:

$$1^{1/4} = \left(e^{i2\pi n}\right)^{1/4} = e^{in\frac{\pi}{2}} = \cos n\frac{\pi}{2} + i\sin n\frac{\pi}{2} \ , n \in Z$$

n=0 $\Rightarrow$ 1+i·0 =1
n=1 $\Rightarrow$ 0+i·1 =i
n=2 $\Rightarrow$ -1+i·0 =-1
n=3 $\Rightarrow$ 0+i·(-1) =-i

We will now calculate 1^i:

$$1^i = \left(e^{i2\pi n}\right)^i = e^{-2\pi n} = \cos(-2\pi n) + i\sin(-2\pi n) = 1$$