INTEGRAL

1. Integral definition
  1.1. Deriving integral for function f(t)=t
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1. Integral definition

Intergral of function f(t) is defined as folows:

$$\int_{t=t_s}^{t=t_e} f(t)dt \tag{1.1}$$

Formula (1.1) is more often written in shorter form, like this:

$$\int_{t_s}^{t_e} f(t)dt \tag{1.2}$$

where it is presumed that bounderies t_e and t_s are connected to the variable which folows after d, in this case this is variable t. Such integral is acctualy a shorter form of writting folowing expression:

$$\int_{t_s}^{t_e} f(t)dt = \sum_{k=0}^{\frac{t_s - t_e}{dt}-1} f(t+kdt)dt \tag{1.3}$$

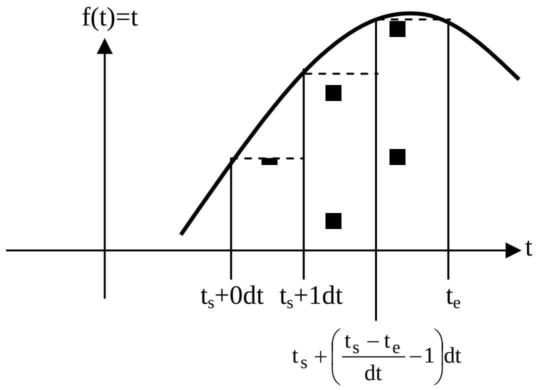
Formula (1.3) can be also written like this:

$$\int_{t_s}^{t_e} f(t)dt = f(t+0dt)dt + f(t+1dt)dt + f(t+2dt)dt + ... + f\left[t + \left(\frac{t_e - t_s}{dt} - 1\right)dt\right]dt \tag{1.4}$$

That means that integral calculates surface betwen the function and horizontal axis. Symbol dt is almost equal to zero because it is defined like this:

$$dt = \frac{1}{\infty}$$

If for dt some finite number is used, then above formula would give aproksimation of the real surface. Formula (1.4) is ilustrated with the folowing picture:



We can check the above formula by presuming that:

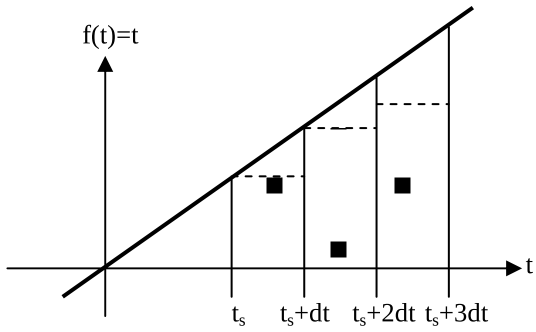
t_s=3; t_e=9; dt=2

and inserting into formula, after which we get that the postition of the last summand should be:

$$t_s + \left(\frac{t_e - t_s}{dt} - 1\right)dt = 3 + \left(\frac{9-3}{2} - 1\right)2 = 3 + 4 = 7$$

1.1. Deriving integral for function f(t)=t

In this chapter integral will be calculated for the function f(t)=t shown in folowing picture:



Using definition (1.3) for integral we can write:

$$\int_{t_s}^{t_e} t dt = (t_s + 0dt)dt + (t_s + 1dt)dt + (t_s + 2dt)dt + ... + \left(t_s + \left(\frac{t_e - t_s}{dt} - 1\right)dt\right)dt$$

By taking out common parameter dt we get:

$$\int_{t_s}^{t_e} t dt = \left[(t_s + 0dt) + (t_s + 1dt) + (t_s + 2dt) + ... + \left(t_s + \left(\frac{t_e - t_s}{dt} - 1\right)dt\right)\right]dt$$

Sum of all t can be calculated as $(t_e\text{-}t_s)t_s/dt$. Using this we get:

$$\int_{t_s}^{t_e} t dt = \left[\frac{t_e - t_s}{dt} t_s + 0dt + 1dt + 2dt + ... + \left(\frac{t_e - t_s}{dt} - 1\right)dt\right]dt$$

By taking out comon parameter dt of the summands left from $(t_e\text{-}t_s)t_s/dt$, we get:

$$\int_{t_s}^{t_e} t dt = \left\{\frac{t_e - t_s}{dt} t_s + \left[0 + 1 + 2 + ... + \left(\frac{t_e - t_s}{dt} - 1\right)\right]dt\right\}dt$$

Using formula for calculating sum(1.3), second part of the sum inside curly brackets can be replaced as folows::

$$\int_{t_s}^{t_e} t dt = \left\{\frac{t_e - t_s}{dt} t_s + \frac{0 + \frac{t_e - t_s}{dt} - 1}{2} \frac{t_e - t_s}{dt} dt\right\}dt$$

By reoranging second summand we get:

$$\int_{t_s}^{t_e} t dt = \left\{\frac{t_e - t_s}{dt} t_s + \frac{1}{2} \frac{t_e - t_s - dt}{dt} \frac{t_e - t_s}{dt} dt\right\}dt$$

Concatunating dt we get:

$$\int_{t_s}^{t_e} t dt = (t_e - t_s)t_s + \frac{1}{2}(t_e - t_s - dt)(t_e - t_s)$$

Second summand can be written as:

$$\int_{t_s}^{t_e} t dt = (t_e - t_s)t_s + \frac{1}{2}\left[(t_e - t_s)^2 - dt(t_e - t_s)\right]$$

Second summand inside square brackets is zero so it can be omitted:

$$\int_{t_s}^{t_e} t dt = (t_e - t_s)t_s + \frac{1}{2}(t_e - t_s)^2$$

Using the formula for second power of summ (2.2) we get:

$$\int_{t_s}^{t_e} t dt = \left(t_e t_s - t_s^2\right) + \frac{1}{2}\left(t_e^2 + t_s^2 - 2t_e t_s\right)$$

Putting everything under same fraction we get:

$$\int_{t_s}^{t_e} t dt = \frac{2t_e t_s - 2t_s^2 + t_e^2 + t_s^2 - 2t_e t_s}{2}$$

Concatunating we get:

$$\int_{t_s}^{t_e} t dt = \frac{t_s^2 - t_e^2}{2}$$

This can be written as:

$$\int_{t_s}^{t_e} t dt = \frac{t^2}{2} \Big/_{t_s}^{t_e}$$

1.2. Formulas used to derive integral for function f(t)=t

$$\sum_{k=0}^{n} (a + kq) + = (a + 0q) + (a + 1q) + (a + 2q) + ... + (a + nq) = \frac{(a + 0q) + (a + nq)}{n}(n + 1) \qquad (2.1)$$

$$(a + b)^2 = a^2 + b^2 + 2ab \qquad (2.2)$$