CLI - Contiuous Laplace Integral - Part II

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1. Laplace tranformations of basic functions

1.1. Laplace transformation of exponential function

- In this chapter we will find Laplace transformation of exponential function defined as:

$$f(t) = \begin{cases} 0 & \text{for } t < 0 \\ e^t & \text{for } t > 0 \end{cases} \qquad (1.1)$$

which is ilustrated on following figure:

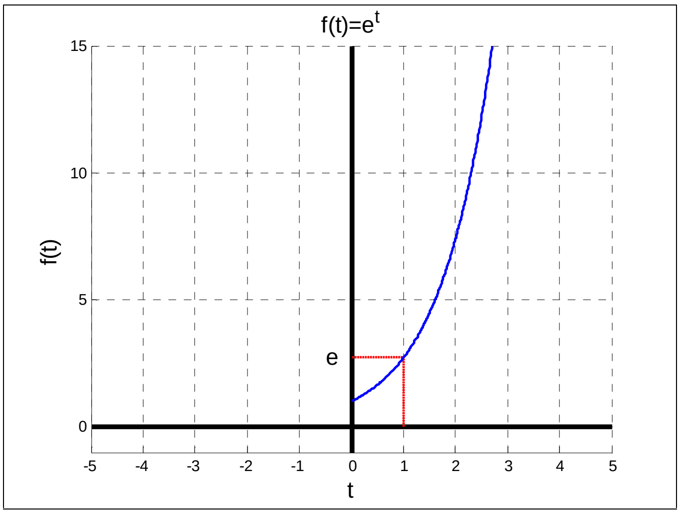
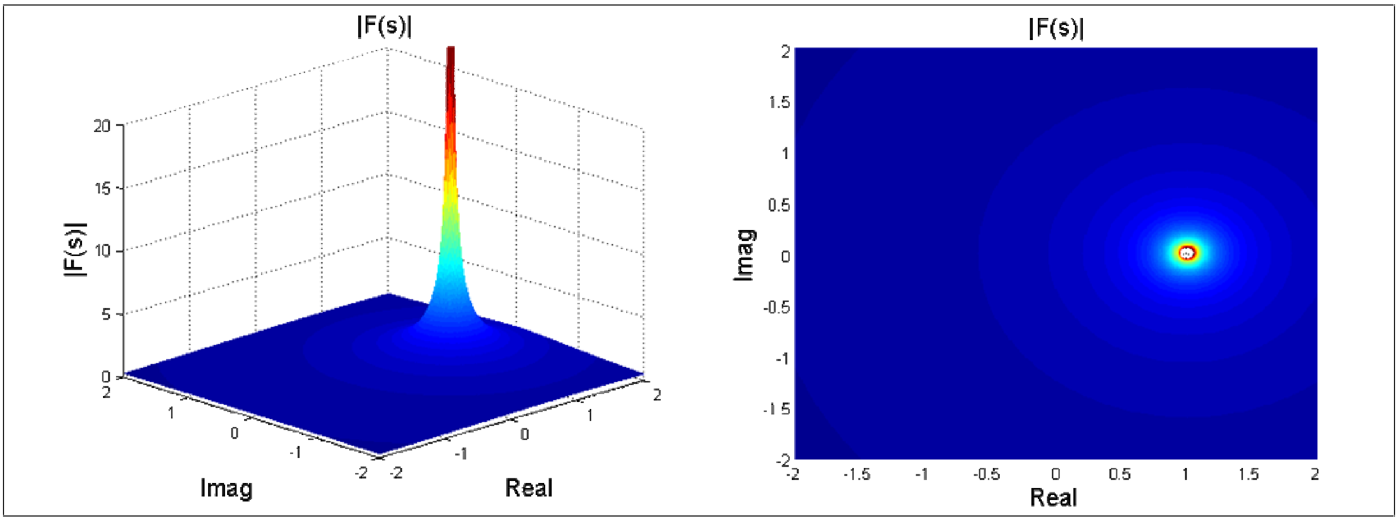


Figure 1.1. Exponential function.

- Laplace transformation is then:

$$F(s) = \frac{1}{s-1} \quad , \quad \sigma \geq 1 \qquad (1.2)$$

and it's magnitude plot looks like this:



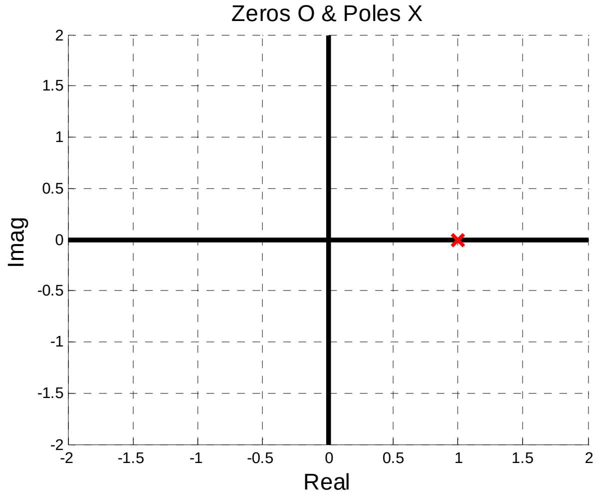


Figure 1.2. Magnitude plot.

- Laplace transformation of exponential function

– We start with the definition of Laplace transformation:

$$F(t) = \int_0^\infty e^{-st} f(t) dt \tag{1.3}$$

– Inserting (1.1) into (1.3) we get:

$$F(t) = \int_0^\infty e^{-st} e^{t} dt$$

$$= \int_0^\infty e^{(1-s)t} dt$$

$$= \frac{1}{1-s} e^{(1-s)t} \Big/_0^\infty$$

$$= \frac{1}{1-s}\left[\lim_{t\to\infty} e^{(1-s)t} - e^{(1-s)0}\right]$$

$$= \frac{1}{1-s}\left[\lim_{t\to\infty} e^{(1-s)t} - 1\right] \tag{1.4}$$

– Using definition for s:

$$s = \sigma + i\omega$$

limes from (1.4) becomes:

$$\lim_{t\to\infty} e^{(1-s)t} = \lim_{t\to\infty} e^{(1-\sigma-i\omega)t}$$

$$= \lim_{t\to\infty} e^{(1-\sigma)t} e^{-i\omega t}$$

$$= \lim_{t\to\infty} \frac{e^{(1-\sigma)t}}{e^{i\omega t}}$$

– For:

$$\sigma = 1$$

(6.2) becomes:

$$\lim_{t\to\infty} e^{(1-s)t} = \lim_{t\to\infty} \frac{e^{0t}}{e^{i\omega t}}$$

$$= \lim_{t\to\infty} \frac{1}{e^{i\omega t}}$$

$$= 0 \qquad , \quad \sigma=1 \tag{1.5}$$

– For

$$\sigma > 1$$

(6.2) exponent in numeroator is negative so we have:

$$\lim_{t\to\infty} e^{(1-s)t} = \lim_{t\to\infty} \frac{e^{-(\sigma-1)t}}{e^{i\omega t}}$$

$$= \lim_{t\to\infty} \frac{}{e^{(\sigma-1)t} e^{i\omega t}}$$

$$= 0 \qquad , \quad \sigma\geq 1 \tag{1.6}$$

– Using (1.5) and (1.6), (1.4) can now be written like this:

$$F(t) = \frac{1}{1-s}(0-1)$$

$$= \frac{1}{s-1} \quad , \quad \sigma \geq 1 \tag{1.7}$$

- Observations

– Condition $\sigma \geq 1$, in order for Laplace Transformation to exist, is result of the fact that only if we multiply e^{t} with $e^{-\sigma t}$ where $\sigma \geq 1$ will get resulting function which doesn't go toward infinity.
– If we choose $\sigma=1$ we get constant function, and for $\sigma>1$ function goes toward zero as demonstrated on following figure:

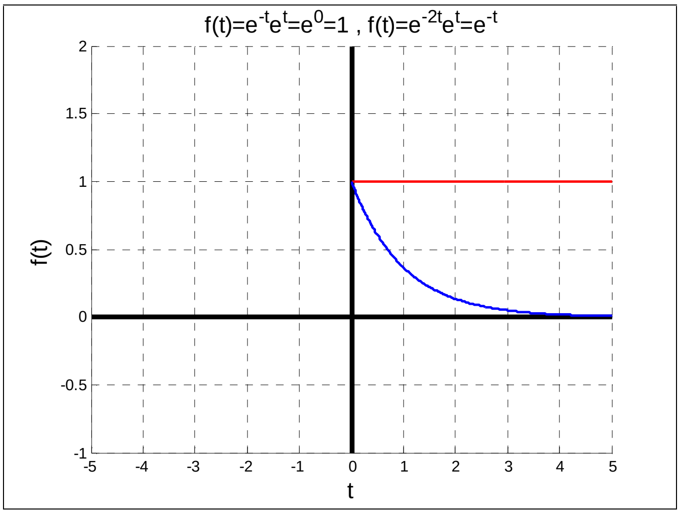


Figure 1.3. Controlling exponential function with $e^{-\sigma t}$.

2. MATLAB

2.1. Figure 1.1.

```
clear;

%DEFINE f(t).
e = 2.718281828;
t =[0:0.01:5];
ft=e.^t;

%PREPARE FIGURE.
figure(1); clf; grid on; hold on; axis([-5 5 -1 15]);
title('f(t)=e^t','FontSize',14); xlabel('t','FontSize',14);
ylabel('f(t)','FontSize',14);
line([-5 5],[0 0],'Color','k','LineWidth',3);
line([0 0],[-1 15],'Color','k','LineWidth',3);

%DRAW e.

text(-0.5,e,'e','FontSize',14)
line([0 1],[e e],'Color','r','LineWidth',2,'LineStyle','--');
line([1 1],[0 e],'Color','r','LineWidth',2,'LineStyle','--');

%DRAW f(t).
plot(t,ft,'LineWidth',2);
```

2.2. Figure 1.2.

```
clear;

%DEFINE f(t).
e = 2.718281828;
t =[0:0.01:5];

%% Sigma=1

sigma=1;
ft=e.^-(sigma*t).*e.^t;

%PREPARE FIGURE.
figure(1); clf; grid on; hold on; axis([-5 5 -1 2]);
title('f(t)=e^{-t}e^t=e^0=1 , f(t)=e^{-2t}e^t=e^{-t}','FontSize',14); xlabel('t','FontSize',14);
ylabel('f(t)','FontSize',14);
line([-5 5],[0 0],'Color','k','LineWidth',3);
line([0 0],[-1 2],'Color','k','LineWidth',3);

%DRAW f(t).
plot(t,ft,'r','LineWidth',2);

sigma=2;
ft=e.^-(sigma*t).*e.^t;
plot(t,ft,'LineWidth',2);
```

2.3. Figure 1.3.

```
%% Define grid.
clear;
rr    = -2:0.01:2;
ii    = -2:0.01:2;
[r,i] = meshgrid(rr,ii);
c     = r + j*i;

%% Transfer function.
num  = [ 1 ];
den  = [ 1 -1 ];
z    = polyval(num,c)./polyval(den,c);

%% Amplituge.
figure(1); clf; grid on; hold on;

axis([-2 2 -2 2 0 20]);
caxis([0 20]);
surf(rr,ii,abs(z));

shading flat;
xlabel('Real','FontSize',15);
ylabel('Imag','FontSize',15);
zlabel('|F(s)|','FontSize',15);
title('|F(s)|','FontSize',15);
view([-44,24]);

%% Zeros & Poles.

%PREPARE FIGURE.
figure(2); clf; grid on; hold on; axis([-2 2 -2 2]);
title('Zeros O & Poles X','FontSize',14);
xlabel('Real','FontSize',14); ylabel('Imag','FontSize',14);
line([-2 2],[0 0],'Color','k','LineWidth',3);
line([0 0],[-2 2],'Color','k','LineWidth',3);

%plot(1,0,'ro','MarkerSize',12,'LineWidth',2.5);
plot(1,0,'rx','MarkerSize',12,'LineWidth',2.5);
```

3. References