DFS - Descrete Fourier Sequence – Part II

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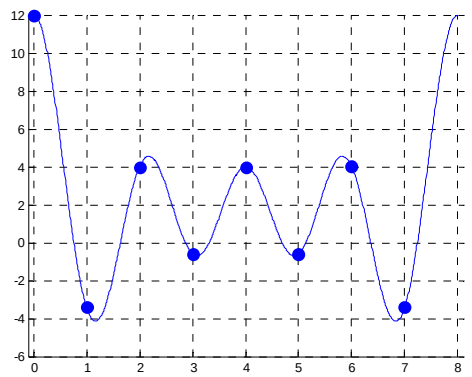
1. How many coefficients to calculate?

– In this chapter we shall try to show how many coefficients should be calculated.

– In following subchapters we will show that high frequency signals get mapped into low frequency ones.
– This means that if you take N samples from one period of signal then all signal components with kω frequences greater then N/2-1 for even N, and (N-1)/2 for odd N are mapped into one of those lower frequence components.

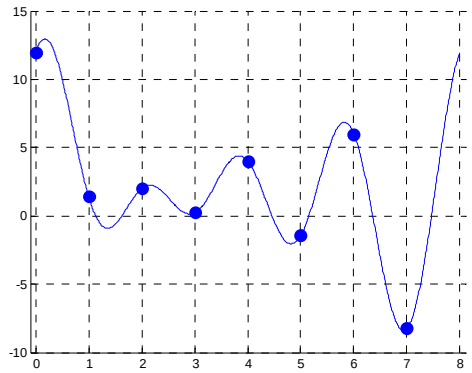
– This is not the case only for component of frequency N/2k when N is even, in our case that would be 4ω when N=8.
– Figure 2.1. shows that a_4 can be calculated but its value will be twice the amplituded of 4cos(4ωt).
– Figure 2.2. shows that a_4 can be detected even if all other detectible components are present in the signal.
– Since b_k can't be calculated and a_4 isn't amplitude of corresponding signal, like all other coefficients are but twice the amplitude value, a_4 is not calculated.



$f(t) = 2+\cos(\omega t)+2\cos(2\omega t)+3\cos(3\omega t)+4\cos(4\omega t)$

$a_0=2,\ a_1=1,\ a_2=2,\ a_3=3,\ a_4=8$

Figure 2.1.



$f(t) = 2+\cos(\omega t)+2\cos(2\omega t)+3\cos(3\omega t)+4\cos(4\omega t)$
$+\sin(\omega t)+2\sin(2\omega t)+3\sin(3\omega t)$
$a_0=2,\quad a_1=1,\ a_2=2,\ a_3=3,\ a_4=8$
$b_1=1,\ b_2=2,\ b_3=3$

Figure 2.2.

– This is why coefficients are calculated only till k=N/2-1 for even N, and k=(N-1)/2 for odd N

- **Comments on calculating DFS coefficients**

– For instance figure 2.3 a) shows that if add component cos(7ωt) to the f(t) this would have the same efect as adding cos(1ωt) since N samples of cos(7ωt) have exactly the same values as N samples of signal cos(1ωt).

– On figure 2.3 e) we see that if we add component cos(8ωt) to f(t) it would have the same efect as adding costant signal 1 since taking N samples of cos(8ωt) gives exactly the same values as taking N samples of constant signal of value 1.

– This is ilustrated with figures 2.4. b) and f) where we see that if we have signal 3cos(2ωt) and we add component 1cos(5ωt) this will have the same effect as adding component 1cos(2ωt) therefore resulting in 4cos(2ωt).

– This is ilustrated with figures 2.6. c) and f) where we see that if we have signal 3cos(3ωt) and we add component 2sin(4ωt) this will have the same effect as adding component -2sin(3ωt) therefore resulting in 1cos(3ωt).

1.1. Examples

1.1.1. Examples of calculating a_k when N=8

– In this chapter we shall calculate a_k coefficients for cosine signals of different frequences as listed in following table while taking N=8 samples per period:

N=8			N/2						N	3N/2							2N
	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}
						a_3	a_2	a_1	$2a_0$	a_1	a_2	a_3	a_4	a_3	a_2	a_1	$2a_0$
2+1cos(1ωt)	2	1	0	0	0	0	0	1	4	1	0	0	0	0	0	1	4
2+2cos(2ωt)	2	0	2	0	0	0	2	0	4	0	2	0	0	0	2	0	4
2+3cos(3ωt)	2	0	0	3	0	3	0	0	4	0	0	3	0	3	0	0	4
2+4cos(4ωt)	2	0	0	0	8	0	0	0	4	0	0	0	8	0	0	0	4
2+3cos(5ωt)	2	0	0	3	0	3	0	0	4	0	0	3	0	3	0	0	4
2+2cos(6ωt)	2	0	2	0	0	0	2	0	4	0	2	0	0	0	2	0	4
2+1cos(7ωt)	2	1	0	0	0	0	0	1	4	1	0	0	0	0	0	1	4
2+3cos(8ωt)	2+3	0	0	0	0	0	0	0	2*5	0	0	0	0	0	0	0	2*5
2+1cos(9ωt)	2	1	0	0	0	0	0	1	4	1	0	0	0	0	0	1	4

– Some of this signals are shown in the following figure:

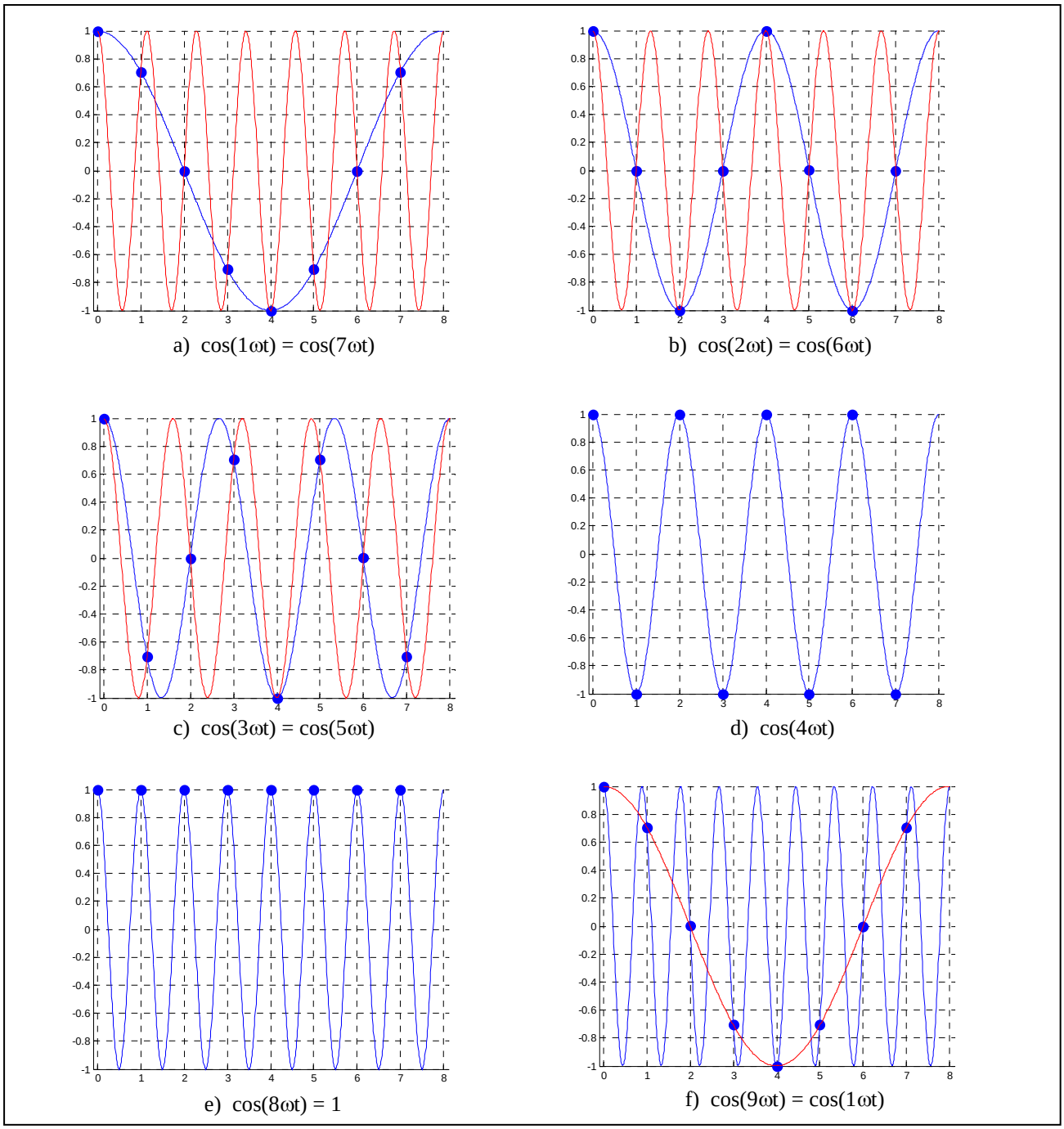


Figure 2.3. Signals of higher frequences are detected as low frequence signals.

1.1.2. Examples of calculating a_k when N=7

– In this chapter we shall calculate a_k coefficients for cosine signals of different frequences as listed in following table while taking N=7 samples per period:

N=7								N							2N
	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}
					a_3	a_2	a_1	$2a_0$	a_1	a_2	a_3	a_3	a_2	a_1	$2a_0$
2+1cos(1ωt)	2	1	0	0	0	0	1	4	1	0	0	0	0	1	4
2+2cos(2ωt)	2	0	2	0	0	2	0	4	0	2	0	0	2	0	4
2+3cos(3ωt)	2	0	0	3	3	0	0	4	0	0	3	3	0	0	4
2+3cos(4ωt)	2	0	0	3	3	0	0	4	0	0	3	3	0	0	4
2+2cos(5ωt)	2	0	2	0	0	2	0	4	0	2	0	0	2	0	4
2+1cos(6ωt)	2	1	0	0	0	0	1	4	1	0	0	0	0	1	4
2+3cos(7ωt)	2	0	0	0	0	0	0	4	0	0	0	0	0	0	4
2+1cos(8ωt)	2	1	0	0	0	0	1	4	1	0	0	0	0	1	4

– Some of this signals are shown in the following figure:

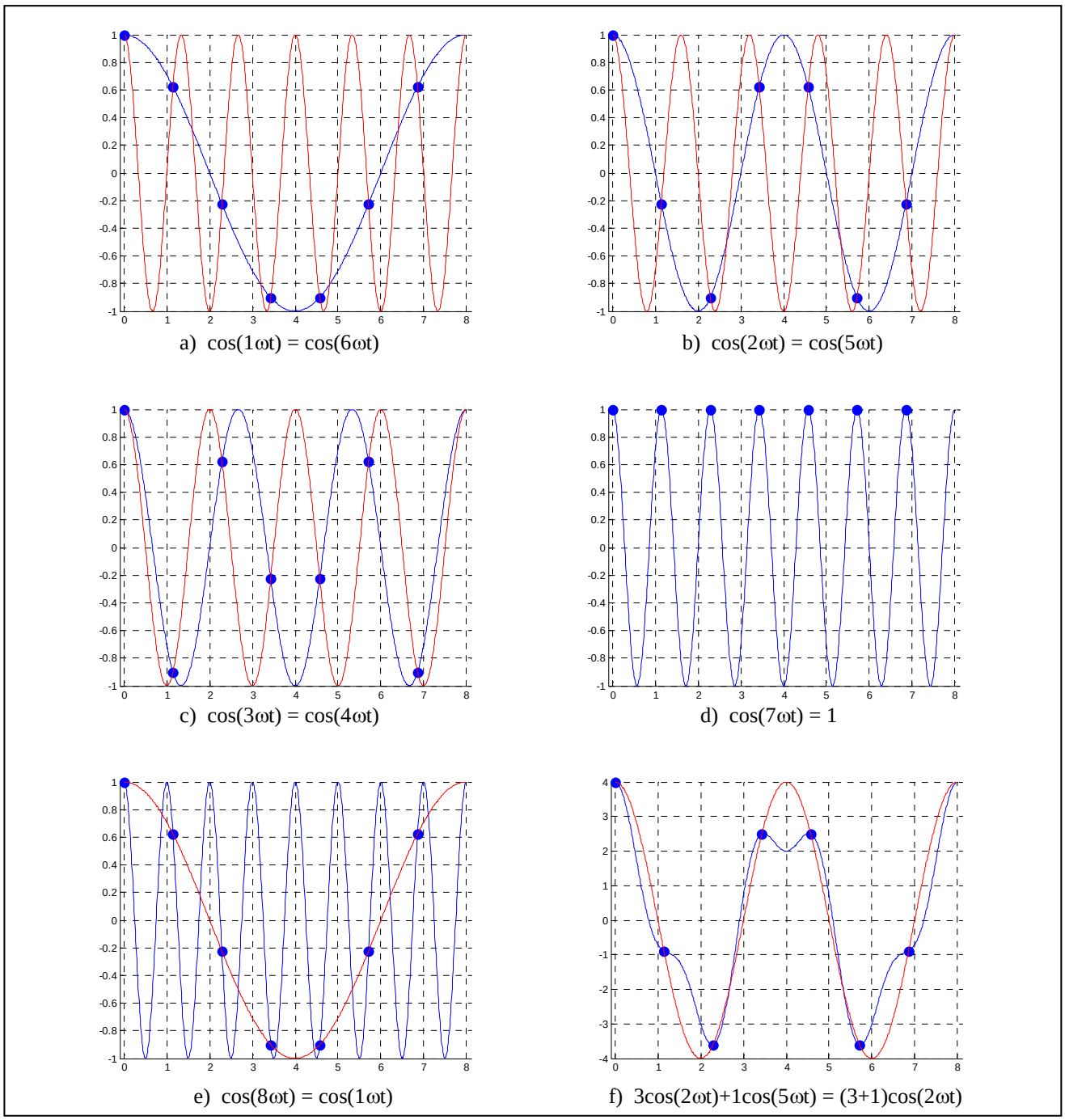


Figure 2.4. Signals of higher frequences are detected as low frequence signals.

1.1.3. Examples of calculating b_k when N=8

– In this chapter we shall calculate b_k coefficients for sine signals of different frequences as listed in following table while taking N=8 samples per period:

N=8					N/2				N				3N/2			
	b_0	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	b_{11}	b_{12}	b_{13}	b_{14}	b_{15}
					0	b_3	b_2	b_1	0	b_1	b_2	b_3	0	b_3	b_2	b_3
2+1in(1ωt)		1	0	0	0	0	0	-1	0	1	0	0	0	0	0	-1
2+2in(2ωt)		0	2	0	0	0	-2	0	0	0	2	0	0	0	-2	
2+3sin(3ωt)		0	0	3	0	-3	0	0	0	0	0	3	0	-3	0	
2+4in(4ωt)		0	0	0	0	0	0	0	0	0	0	0	0	0	0	
2+3sin(5ωt)		0	0	-3	0	3	0	0	0	0	0	-3	0	3	0	
2+2in(6ωt)		0	-2	0	0	0	2	0	0	0	-2	0	0	0	2	
2+1in(7ωt)		-1	0	0	0	0	0	1	0	-1	0	0	0	0	0	1
2+3sin(8ωt)		0	0	0	0	0	0	0	0	0	0	0	0	0	0	
2+1sin(9ωt)		1	0	0	0	0	0	-1	0	1	0	0	0	0	0	-1

– Some of this signals are shown in the following figure:

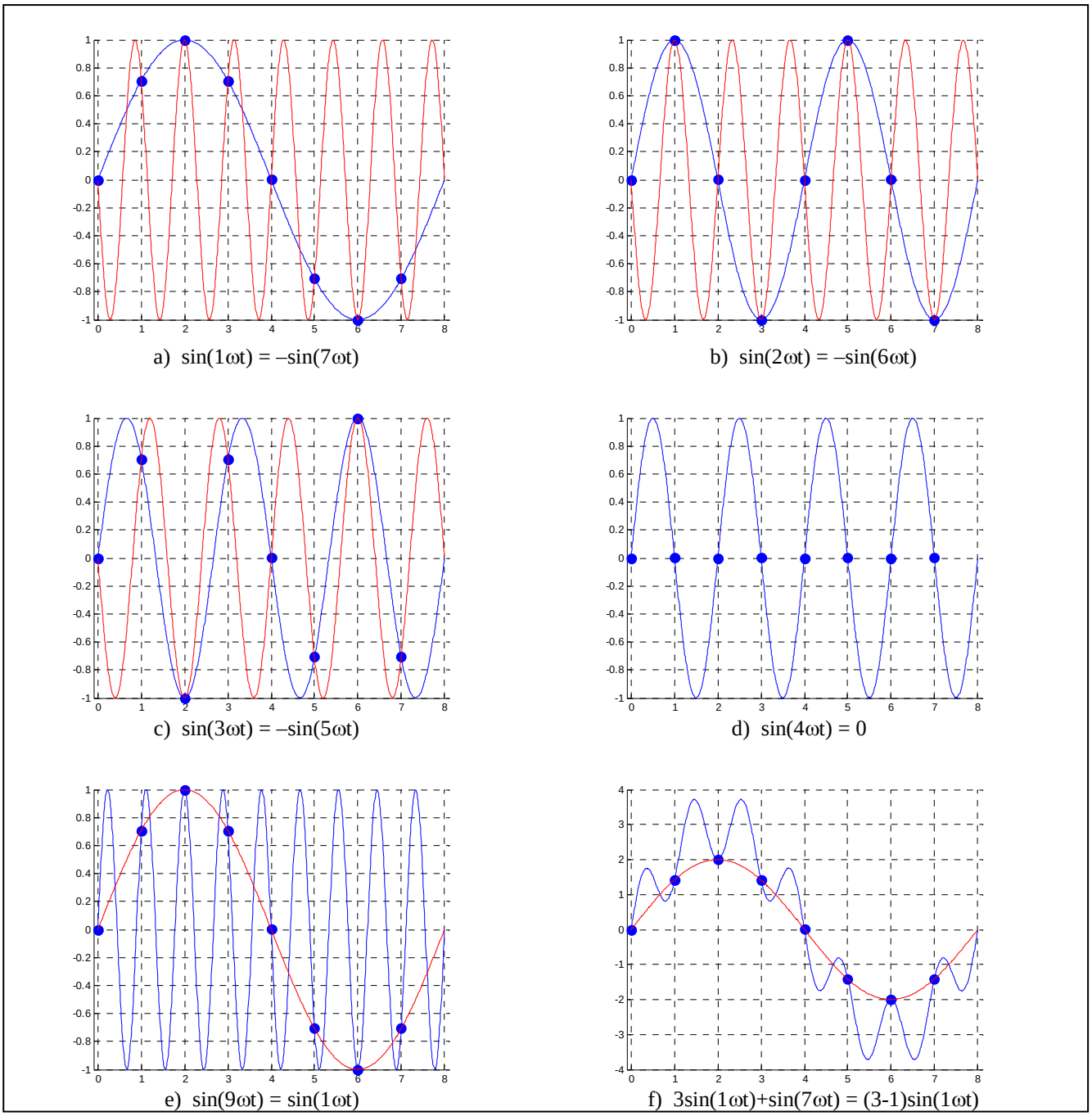


Figure 2.5 Signals of higher frequences are detected as low frequence signals.

1.1.4. Examples of calculating b_k when N=7

– In this chapter we shall calculate b_k coefficients for sine signals of different frequences as listed in following table while taking N=7 samples per period:

N=7								N							2N
	b_0	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	b_{11}	b_{12}	b_{13}	b_{14}
					$-b_3$	$-b_2$	$-b_1$	$2b_0$	b_1	b_2	b_3	$-b_3$	$-b_2$	$-b_1$	$2b_0$
2+1sin(1ωt)		1	0	0	0	0	-1	0	1	0	0	0	0	-1	0
2+2sin(2ωt)		0	2	0	0	-2	0	0	0	2	0	0	-2	0	0
2+3sin(3ωt)		0	0	3	-3	0	0	0	0	0	3	-3	0	0	0
2+3sin(4ωt)		0	0	-3	3	0	0	0	0	0	-3	3	0	0	0
2+2sin(5ωt)		0	-2	0	0	2	0	0	0	-2	0	0	2	0	0
2+1sin(6ωt)		-1	0	0	0	0	1	0	-1	0	0	0	0	1	0
2+3sin(7ωt)		0	0	0	0	0	0	0	0	0	0	0	0	0	0
2+1sin(8ωt)		1	0	0	0	0	-1	0	1	0	0	0	0	-1	0

– Some of this signals are shown in the following figure:

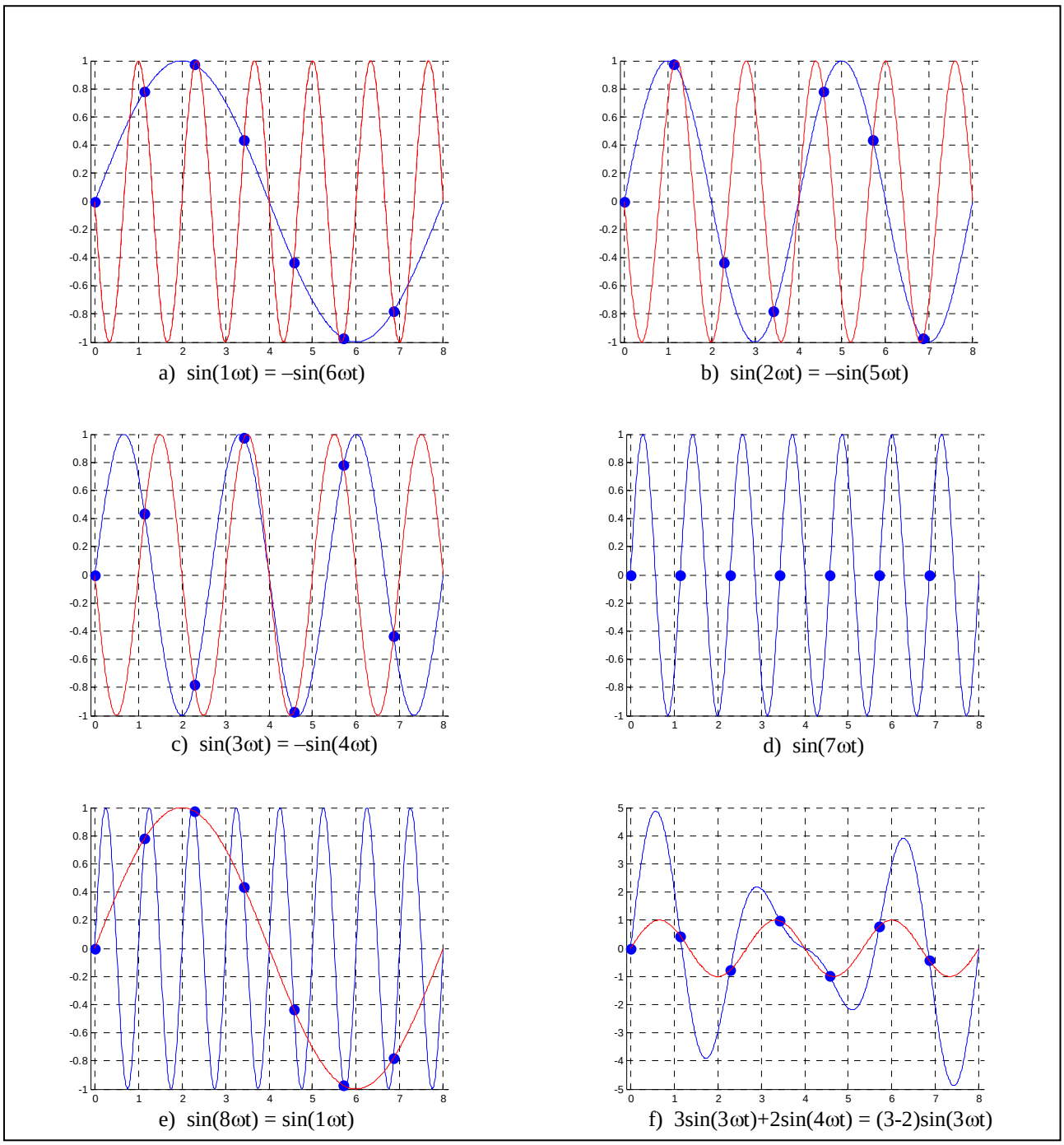


Figure 2.6. Signals of higher frequences are detected as low frequence signals.

1.2. Proofs

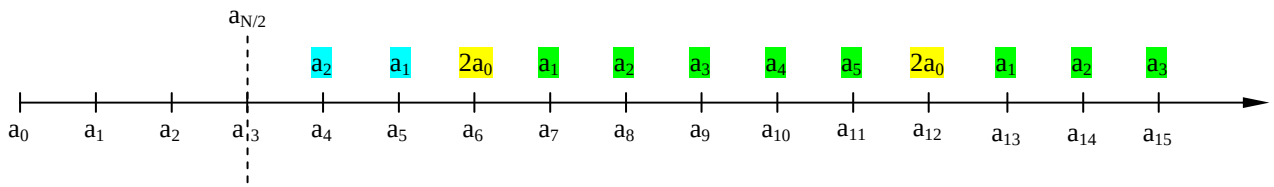
– Calculating a_k.

$$N = 6 \tag{2.1}$$

$$a_{\ell N} = 2a_0 \tag{2.2}$$

$$a_{\ell N+k} = a_k \tag{2.3}$$

$$a_{N/2+k} = a_{N/2-k} \tag{2.4}$$



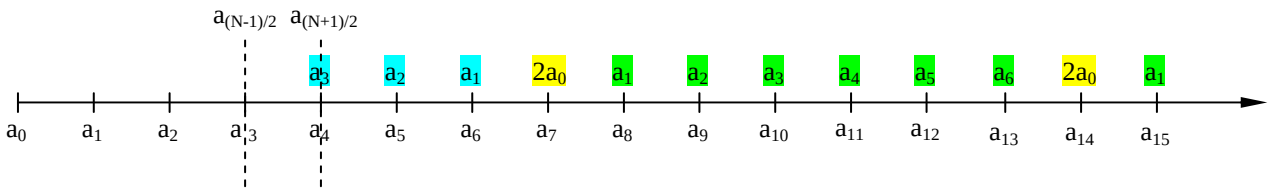
– Calculating a_k.

$$N = 7 \tag{2.5}$$

$$a_{\ell N} = 2a_0 \tag{2.6}$$

$$a_{\ell N+k} = a_k \tag{2.7}$$

$$a_{(N+1)/2+k} = a_{(N-1)/2-k} \tag{2.8}$$



– Calculating b_k.

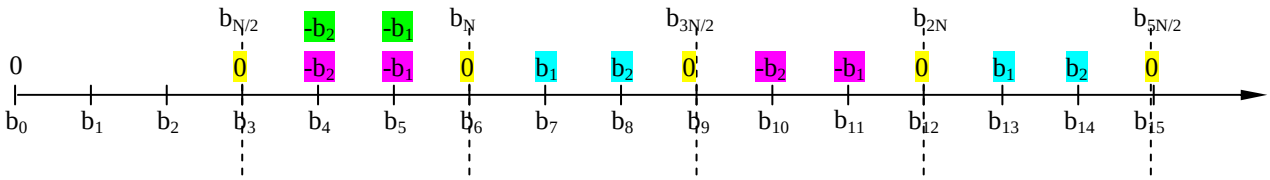
$$N = 6 \tag{2.9}$$

$$b_{\ell N/2} = 0 \qquad \text{where } \ell = 0,1,2,... \tag{2.10}$$

$$b_{\ell N+k} = b_k \tag{2.11}$$

$$b_{\ell N-k} = -b_k \tag{2.12}$$

$$b_{\ell N/2+k} = -b_{\ell N/2-k} \qquad \text{where } \ell = 0,1,2,... \tag{2.13}$$



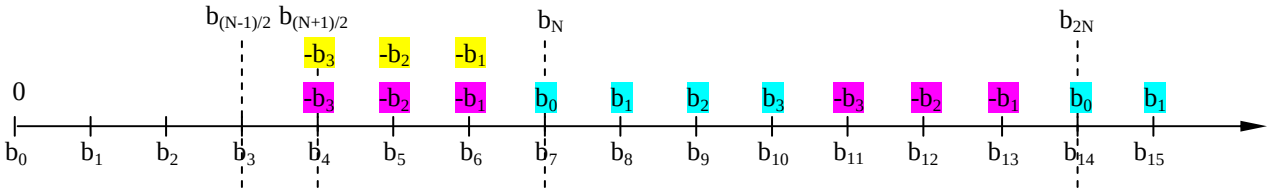
– Calculating b_k.

$$N = 7 \tag{2.14}$$

$$b_{(N+1)/2+k} = -b_{(N-1)/2-k} \tag{2.15}$$

$$b_{\ell N+k} = b_k \tag{2.16}$$

$$b_{\ell N-k} = -b_k \tag{2.17}$$



1.2.1. Proof for $a_{lN} = 2a_0$

- In this chapter we will show that there is no point in calculating:

$$a_{\ell N} \text{ where } \ell = 1,2,...$$

because $a_{\ell N}$ gives us now new information since:

$$a_{\ell N} = 2a_0 \quad (2.18)$$

- Rewriting (3.15):

$$a_k = \frac{2}{N}\sum_{n=1}^{N} f(n)\cos(k\psi n)$$

for $k = \ell N$ we get:

$$a_{\ell N} = \frac{2}{N}\sum_{n=1}^{N} f(n)\cos(\ell N\psi n) \quad (2.19)$$

$$= \frac{2}{N}\sum_{n=1}^{N} f(n)\cos\left(\ell N\frac{2\pi}{N}n\right)$$

$$= \frac{2}{N}\sum_{n=1}^{N} f(n)\cos(\ell 2\pi n)$$

$$= \frac{2}{N}\sum_{n=1}^{N} f(t)\cdot 1$$

$$= \frac{2}{N}\sum_{n=1}^{N} f(t)$$

– Inserting (3.14):

$$a_0 = \frac{1}{N}\sum_{n=1}^{N} f(n) \quad (2.20)$$

we get:

$$a_{\ell N} = 2a_0 \quad (2.21)$$

1.2.2. Proof for $a_{lN+k} = a_k$

- In this chapter we will show that there is no point in calculating:

$$a_{\ell N+k} \text{ where } \ell = 1,2,...$$

because $a_{\ell N+k}$ gives us now new information since:

$$a_{\ell N+k} = a_k \tag{2.22}$$

- Rewriting (3.15):

$$a_k = \frac{2}{N}\sum_{n=1}^{N} f(n)\cos(k\psi n) \tag{2.23}$$

for $k = \ell N + k$ we get:

$$a_{\ell N+k} = \frac{2}{N}\sum_{n=1}^{N} f(n)\cos[(\ell N + k)\psi n] \tag{2.24}$$

– Now in order to prove (2.22) we need to prove:

$$\cos(k\psi n) = \cos[(\ell N + k)\psi n] \tag{2.25}$$

$$= \cos(\ell N\psi n + k\psi n) \tag{2.26}$$

$$= \cos\left(\ell N\frac{2\pi}{N}n + k\psi n\right) \tag{2.27}$$

$$= \cos(\ell 2\pi n + k\psi n) \tag{2.28}$$

$$= \cos(\ell 2\pi n)\cos(k\psi n) - \sin(\ell 2\pi n)\sin(k\psi n) \tag{2.29}$$

$$= 1\cdot\cos(k\psi n) - 0\cdot\sin(k\psi n) \tag{2.30}$$

$$= \cos(k\psi n) \tag{2.31}$$

1.2.3. Proof for $a_{N/2+k} = a_{N/2-k}$ for even N

- In this chapter we will show that there is no point in calculating:

$$a_{N/2+k}$$

because $a_{\ell N/2+k}$ gives us now new information since:

$$a_{N/2+k} = a_{N/2-k} \tag{2.32}$$

- We will now rewrite (3.15):

$$a_k = \frac{2}{N}\sum_{n=1}^{N} f(n)\cos(k\psi n) \tag{2.33}$$

for $k = N/2 + k$ and $k = N/2 - k$:

$$a_{N/2+k} = \frac{2}{N}\sum_{n=1}^{N} f(n)\cos\left[\left(\frac{N}{2}+k\right)\psi n\right] \tag{2.34}$$

$$a_{N/2-k} = \frac{2}{N}\sum_{n=1}^{N} f(n)\cos\left[\left(\frac{N}{2}-k\right)\psi n\right] \tag{2.35}$$

– Now in order to prove (2.32) we need to prove:

$$\cos\left[\left(\frac{N}{2}+k\right)\psi n\right] = \cos\left[\left(\frac{N}{2}-k\right)\psi n\right] \tag{2.36}$$

$$\cos\left(\frac{N}{2}\psi n + k\psi n\right) = \cos\left(\frac{N}{2}\psi n - k\psi n\right) \tag{2.37}$$

$$\cos\left(\frac{N}{2}\frac{2\pi}{N}n + k\psi n\right) = \cos\left(\frac{N}{2}\frac{2\pi}{N}n - k\psi n\right) \tag{2.38}$$

$$\cos(\pi n + k\psi n) = \cos(\pi n - k\psi n) \tag{2.39}$$

$$\cos(\pi n)\cos(k\psi n) - \sin(\pi n)\sin(k\psi n) = \cos(\pi n)\cos(-k\psi n) - \sin(\pi n)\sin(-k\psi n) \tag{2.40}$$

$$\cos(\pi n)\cos(k\psi n) - 0\sin(k\psi n) = \cos(\pi n)\cos(-k\psi n) - 0\sin(-k\psi n) \tag{2.41}$$

$$\cos(\pi n)\cos(k\psi n) = \cos(\pi n)\cos(k\psi n) \tag{2.42}$$

$$0 = 0 \tag{2.43}$$

1.2.4. Proof for $a_{(N-1)/2+k} = a_{(N+1)/2-k}$ for even N

- In this chapter we will show that there is no point in calculating $a_{(N+1)/2+k}$ since it gives now new information since:

$$a_{(N+1)/2+k} = a_{(N-1)/2-k} \tag{2.44}$$

- We will now rewrite (3.15):

$$a_k = \frac{2}{N}\sum_{n=1}^{N} f(n)\cos(k\psi n) \tag{2.45}$$

for $k = (N+1)/2 + k$ and $k = (N-1)/2 - k$:

$$a_{(N+1)/2+k} = \frac{2}{N}\sum_{n=1}^{N} f(n)\cos\left[\left(\frac{N+1}{2}+k\right)\psi n\right] \tag{2.46}$$

$$a_{(N-1)/2-k} = \frac{2}{N}\sum_{n=1}^{N} f(n)\cos\left[\left(\frac{N-1}{2}-k\right)\psi n\right] \tag{2.47}$$

- We can now prove that (2.46) and (2.47) are equal using:

$$\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta) \tag{2.48}$$

$$\psi = \frac{2\pi}{N} \tag{2.49}$$

simply by proving:

$$\cos\left[\left(\frac{N+1}{2}+k\right)\psi n\right] = \cos\left[\left(\frac{N-1}{2}-k\right)\psi n\right] \tag{2.50}$$

$$\cos\left(\frac{N+1}{2}\psi n + k\psi n\right) = \cos\left(\frac{N-1}{2}\psi n - k\psi n\right) \tag{2.51}$$

$$\cos\left(\frac{N}{2}\psi n + \frac{1}{2}\psi n + k\psi n\right) = \cos\left(\frac{N}{2}\psi n + \frac{1}{2}\psi n - k\psi n\right) \tag{2.52}$$

$$\cos\left(\frac{N}{2}\frac{2\pi}{N}n + \frac{1}{2}\frac{2\pi}{N}n + k\psi n\right) = \cos\left(\frac{N}{2}\frac{2\pi}{N}n - \frac{1}{2}\frac{2\pi}{N}n - k\psi n\right) \tag{2.53}$$

$$\cos\left(\pi n + \frac{\pi}{N}n + k\psi n\right) = \cos\left(\pi n - \frac{\pi}{N}n - k\psi n\right) \tag{2.54}$$

$$\cos(\pi n)\cos\left(\frac{\pi}{N}n + k\psi n\right) - \sin(\pi n)\sin\left(\frac{\pi}{N}n + k\psi n\right) = \cos(\pi n)\cos\left(-\frac{\pi}{N}n - k\psi n\right) - \sin(\pi n)\sin\left(-\frac{\pi}{N}n - k\psi n\right) \tag{2.55}$$

$$\cos(\pi n)\cos\left(\frac{\pi}{N}n + k\psi n\right) - 0\sin\left(\frac{\pi}{N}n + k\psi n\right) = \cos(\pi n)\cos\left(-\frac{\pi}{N}n - k\psi nn\right) - 0\sin\left(-\frac{\pi}{N}n - k\psi n\right) \tag{2.56}$$

$$\cos(\pi n)\cos\left(\frac{\pi}{N}n + k\psi n\right) = \cos(\pi n)\cos\left(-\frac{\pi}{N}n - k\psi n\right) \tag{2.57}$$

$$\cos(\pi n)\cos\left(\frac{\pi}{N}n + k\psi n\right) = \cos(\pi n)\cos\left(\frac{\pi}{N}n + k\psi n\right) \tag{2.58}$$

$$0 = 0 \tag{2.59}$$

1.2.5. Proof for $b_{lN/2}=0$

- In this chapter we will show that there is no point in calculating:

$$b_{\ell N/2} \text{ where } \ell = 0,1,2,...$$

because:

$$b_{\ell N/2} = 0 \tag{2.60}$$

- Rewriting (3.16):

$$b_k = \frac{2}{N}\sum_{n=1}^{N} f(n)\sin(k\psi n) \tag{2.61}$$

for $k = \ell N/2$ we get:

$$b_{\ell N/2} = \frac{2}{N}\sum_{n=1}^{N} f(n)\sin\left(\ell\frac{N}{2}\psi n\right) \tag{2.62}$$

and using:

$$\psi = \frac{2\pi}{N} \tag{2.63}$$

we get:

$$b_{\ell N/2} = \frac{2}{N}\sum_{n=1}^{N} f(n)\sin\left(\frac{\ell N}{2}\frac{2\pi}{N}n\right) \tag{2.64}$$

$$= \frac{2}{N}\sum_{n=1}^{N} f(n)\sin(\ell\pi n) \tag{2.65}$$

$$= \frac{2}{N}\sum_{n=1}^{N} f(n)\cdot 0 \tag{2.66}$$

$$= 0 \tag{2.67}$$

1.2.6. Proof for $b_{N/2+k} = -b_{N/2-k}$

- In this chapter we will show that there is no point in calculating:

$$b_{\ell N/2+k} \text{ where } \ell = 0,1,2,...$$

because:

$$b_{\ell N/2+k} = -b_{\ell N/2-k} \tag{2.68}$$

- Rewriting (3.16):

$$b_k = \frac{2}{N}\sum_{n=1}^{N} f(n)\sin(k\psi n) \tag{2.69}$$

for $k = \ell N/2 + k$ and $k = \ell N/2 - k$ we get:

$$b_{\ell N/2+k} = \frac{2}{N}\sum_{n=1}^{N} f(n)\sin\left[\left(\ell\frac{N}{2}+k\right)\psi n\right] \tag{2.70}$$

$$b_{\ell N/2-k} = -\frac{2}{N}\sum_{n=1}^{N} f(n)\sin\left[\left(\ell\frac{N}{2}-k\right)\psi n\right] \tag{2.71}$$

- We can now prove that (2.70) and (2.71) are equal using:

$$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta)+\cos(\alpha)\sin(\beta) \tag{2.72}$$

$$\psi = \frac{2\pi}{N} \tag{2.73}$$

simply by proving:

$$\sin\left[\left(\ell\frac{N}{2}+k\right)\psi n\right] = -\sin\left[\left(\ell\frac{N}{2}-k\right)\psi n\right] \tag{2.74}$$

$$\sin\left(\ell\frac{N}{2}\psi n + k\psi n\right) = -\sin\left(\ell\frac{N}{2}\psi n - k\psi n\right) \tag{2.75}$$

$$\sin\left(\ell\frac{N}{2}\frac{2\pi}{N}n + k\psi n\right) = -\sin\left(\ell\frac{N}{2}\frac{2\pi}{N}n - k\psi n\right) \tag{2.76}$$

$$\sin(\ell\pi n + k\psi n) = -\sin(\ell\pi n - k\psi n) \tag{2.77}$$

$$\sin(\ell\pi n)\cos(k\psi n)+\cos(\ell\pi n)\sin(k\psi n) = -[\sin(\ell\pi n)\cos(-k\psi n)+\cos(\ell\pi n)\sin(-k\psi n)] \tag{2.78}$$

$$0\cos(k\psi n)+\cos(\ell\pi n)\sin(k\psi n) = -[0\cos(-k\psi n)+\cos(\ell\pi n)\sin(-k\psi n)] \tag{2.79}$$

$$\cos(\ell\pi n)\sin(k\psi n) = -\cos(\ell\pi n)\sin(-k\psi n) \tag{2.80}$$

$$\cos(\ell\pi n)\sin(k\psi n) = \cos(\ell\pi n)\sin(k\psi n) \tag{2.81}$$

$$0 = 0 \tag{2.82}$$

1.2.7. Proof for $b_{lN-k} = -b_k$

- In this chapter we will show that there is no point in calculating:

$$b_{\ell N-k} \text{ where } \ell = 1,2,...$$

because:

$$b_{\ell N-k} = -b_k \tag{2.83}$$

- Rewriting (3.16):

$$b_k = \frac{2}{N}\sum_{n=1}^{N} f(n)\sin(k\psi n) \tag{2.84}$$

for $k = \ell N - k$ and $k = k$ we get:

$$b_{\ell N-k} = \frac{2}{N}\sum_{n=1}^{N} f(n)\sin[(\ell N - k)\psi n] \tag{2.85}$$

$$-b_k = -\frac{2}{N}\sum_{n=1}^{N} f(n)\sin(k\psi n) \tag{2.86}$$

- We can now prove that (2.85) and (2.86) are equal using:

$$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta)+\cos(\alpha)\sin(\beta) \tag{2.87}$$

$$\psi = \frac{2\pi}{N} \tag{2.88}$$

simply by proving:

$$-\sin(k\psi n) = \sin[(\ell N - k)\psi n] \tag{2.89}$$

$$= \sin(\ell N\psi n - k\psi n) \tag{2.90}$$

$$= \sin\left(\ell N\frac{2\pi}{N}n - k\psi n\right) \tag{2.91}$$

$$= \sin(\ell 2\pi n - k\psi n) \tag{2.92}$$

$$= \sin(\ell 2\pi n)\cos(-k\psi n)+\cos(\ell 2\pi n)\sin(-k\psi n) \tag{2.93}$$

$$= \sin(\ell 2\pi n)\cos(k\psi n)-\cos(\ell 2\pi n)\sin(k\psi n) \tag{2.94}$$

$$= 0\cos(k\psi n)-1\sin(k\psi n) \tag{2.95}$$

$$= -\sin(k\psi n) \tag{2.96}$$

1.2.8. Proof for $b_{lN+k} = b_k$

- In this chapter we will show that there is no point in calculating:

$$b_{\ell N+k} \text{ where } \ell = 1,2,...$$

because:

$$b_{\ell N+k} = b_k \tag{2.97}$$

- Rewriting (3.16):

$$b_k = \frac{2}{N}\sum_{n=1}^{N} f(n)\sin(k\psi n) \tag{2.98}$$

for $k = \ell N + k$ and $k = k$ we get:

$$b_{\ell N+k} = \frac{2}{N}\sum_{n=1}^{N} f(n)\sin[(\ell N + k)\psi n] \tag{2.99}$$

$$b_k = \frac{2}{N}\sum_{n=1}^{N} f(n)\sin(k\psi n) \tag{2.100}$$

- We can now prove that (2.99) and (2.100) are equal using:

$$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta)+\cos(\alpha)\sin(\beta) \tag{2.101}$$

$$\psi = \frac{2\pi}{N} \tag{2.102}$$

simply by proving:

$$\sin(k\psi n) = \sin[(\ell N + k)\psi n] \tag{2.103}$$

$$= \sin(\ell N\psi n + k\psi n) \tag{2.104}$$

$$= \sin\left(\ell N\frac{2\pi}{N}n + k\psi n\right) \tag{2.105}$$

$$= \sin(\ell 2\pi n + k\psi n) \tag{2.106}$$

$$= \sin(\ell 2\pi n)\cos(k\psi n)+\cos(\ell 2\pi n)\sin(k\psi n) \tag{2.107}$$

$$= \sin(\ell 2\pi n)\cos(k\psi n)+\cos(\ell 2\pi n)\sin(k\psi n) \tag{2.108}$$

$$= 0\cos(k\psi n)+1\sin(k\psi n) \tag{2.109}$$

$$= \sin(k\psi n) \tag{2.110}$$

1.2.9. Proof for $b_{(N-1)/2+k} = b_{(N+1)/2-k}$ for odd N

- In this chapter we will show that there is no point in calculating $b_{(N-1)/2+k}$ since it gives now new information since:

$$b_{(N+1)/2+k} = -b_{(N-1)/2-k} \tag{2.111}$$

- Rewriting (3.16):

$$b_k = \frac{2}{N}\sum_{n=1}^{N} f(n)\sin(k\psi n) \tag{2.112}$$

for $k = (N-1)/2 - k$ and $k = (N+1)/2 + k$ we get:

$$b_{(N+1)/2+k} = \frac{2}{N}\sum_{n=1}^{N} f(n)\sin\left[\left(\frac{N+1}{2}+k\right)\psi n\right] \tag{2.113}$$

$$-b_{(N-1)/2-k} = -\frac{2}{N}\sum_{n=1}^{N} f(n)\sin\left[\left(\frac{N-1}{2}-k\right)\psi n\right] \tag{2.114}$$

- We can now prove that (2.113) and (2.114) are equal using:

$$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta)+\cos(\alpha)\sin(\beta) \tag{2.115}$$

$$\psi = \frac{2\pi}{N} \tag{2.116}$$

simply by proving:

$$\sin\left[\left(\frac{N+1}{2}+k\right)\psi n\right] = -\sin\left[\left(\frac{N-1}{2}-k\right)\psi n\right] \tag{2.117}$$

$$\sin\left(\frac{N+1}{2}\psi n + k\psi n\right) = -\sin\left(\frac{N-1}{2}\psi n - k\psi n\right) \tag{2.118}$$

$$\sin\left(\frac{N}{2}\psi n + \frac{1}{2}\psi n + k\psi n\right) = -\sin\left(\frac{N}{2}\psi n - \frac{1}{2}\psi n - k\psi n\right) \tag{2.119}$$

$$\sin\left(\frac{N}{2}\frac{2\pi}{N}n + \frac{1}{2}\frac{2\pi}{N}n + k\psi n\right) = -\sin\left(\frac{N}{2}\frac{2\pi}{N}n - \frac{1}{2}\frac{2\pi}{N}n - k\psi n\right) \tag{2.120}$$

$$\sin\left(\pi n + \frac{\pi}{N}n + k\psi n\right) = -\sin\left(\pi n - \frac{\pi}{N}n - k\psi n\right) \tag{2.121}$$

$$\sin(\pi n)\cos\left(\frac{\pi}{N}n + k\psi n\right) + \cos(\pi n)\sin\left(\frac{\pi}{N}n + k\psi n\right) = -\sin(\pi n)\cos\left(-\frac{\pi}{N}n - k\psi n\right) - \cos(\pi n)\sin\left(-\frac{\pi}{N}n - k\psi n\right) \tag{2.122}$$

$$0\cos\left(\frac{\pi}{N}n + k\psi n\right) + \cos(\pi n)\sin\left(\frac{\pi}{N}n + k\psi n\right) = -0\cos\left(-\frac{\pi}{N}n - k\psi n\right) - \cos(\pi n)\sin\left(-\frac{\pi}{N}n - k\psi n\right) \tag{2.123}$$

$$\cos(\pi n)\sin\left(\frac{\pi}{N}n + k\psi n\right) = -\cos(\pi n)\sin\left(-\frac{\pi}{N}n - k\psi n\right) \tag{2.124}$$

$$\sin\left(\frac{\pi}{N}n + k\psi n\right) = -\sin\left(-\frac{\pi}{N}n - k\psi n\right) \tag{2.125}$$

$$-\sin\left(\frac{\pi}{N}n + k\psi n\right) = \sin\left(\frac{\pi}{N}n + k\psi n\right) \tag{2.126}$$

$$0 = 0 \tag{2.127}$$

2. Additional observations

- In this set of chapters we will show what happens with Fourier Transformation when not all of the prerequisites are met**.**
- What happens when we don't take exactly one period of observed signal but multiple periods.
- Or what happens when we don't take integer number of periods but for instance period and half.
- What happens when signal we are observing changes over time.

2.1. Observation period contains integer number of signal periods

– We will now demonstrate what happens if samples are not taken only from single period but from more then 1 period.

– Figure 3.1. shows what happens if we feed DFT with samples of two periods.

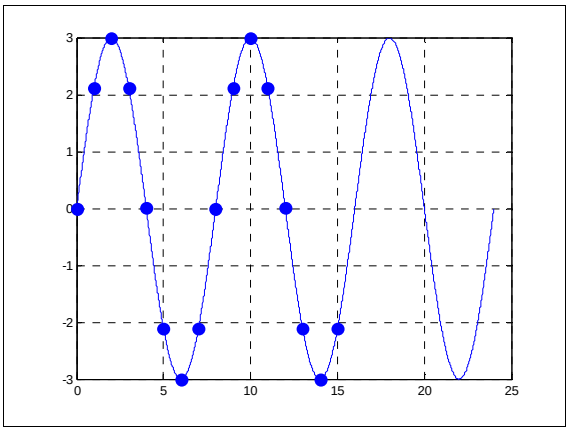


Figure 3.1. $f(t) = 3\sin(\omega t)$, $A_2=3$

– DFT will considere that those samples represent single period which we shall call ω_{DFT} and therefore we have:

$$\omega = \frac{2\pi}{T} \tag{3.1}$$

$$T_{DFT}=2T \tag{3.2}$$

$$\omega_{DFT} = \frac{2\pi}{T_{DFT}} = \frac{2\pi}{2T} = \frac{\pi}{T} \tag{3.3}$$

where:
- T – period of original signal,
- ω – frequency of original signal,
- T_{DFT} – DFT thinks that signal period is 2T,
- ω_{DFT} – DFT thinks that signal base frequency is π/T.

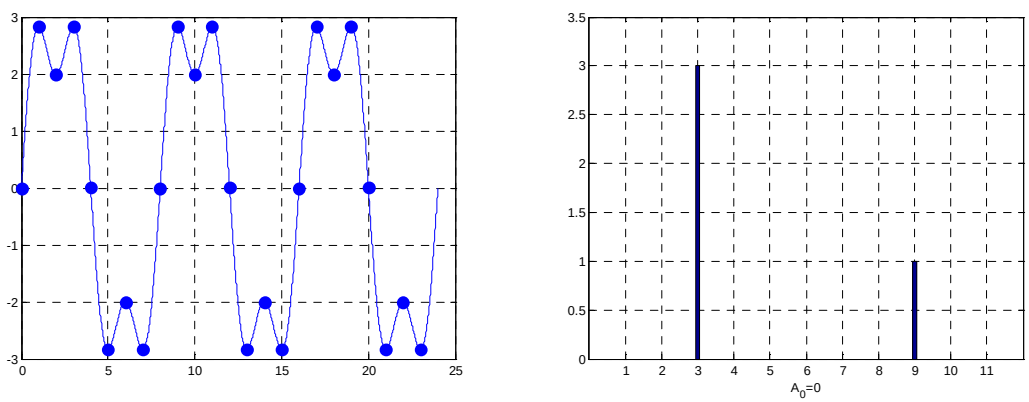
– DFT will not detect sine or cosine component with period $T_{DFT}=2T$, since there is none, therefore $A_1=0$.
– By spitting out $A_2=3$ DFT detects component with frequency two times grater then ω_{DFT}:

$$f(t) = A_2 \sin(2\omega_{DFT} t)$$

– Using (3.1), (3.2) and (3.3) we get the original signal:

$$f(t) = A_2 \sin(2\omega_{DFT} t) = 3\sin(2\frac{\pi}{T} t) = 3\sin(\frac{2\pi}{T} t) = 3\sin(\omega t)$$

– Another example is demonstrated on following figure:



a) $f(t) = 3\sin(\omega t)+\sin(3\omega t)$ b) $A_3=3$, $A_9=1$

Figure 3.2. Feeding DFT with 3 periods results in detecting frequences 3 times higer then originals.

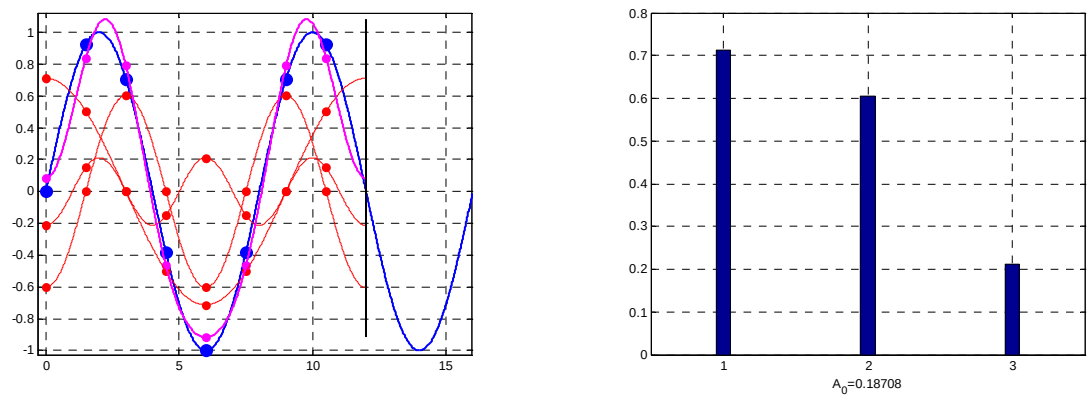
2.2. Spectral leakage

- **What is spectral leakage?**

– Spectral leakage occurs when observation period contains non integer number of signal periods [2].
– DFT then detects frequences which don't exist in original signal but are around that frequency so it is like that frequency has leaked it's energy to surrounding frequences.
– This happens because there is non integere relation between periods of original signal and periods of sine signals which DFT uses to aproximate original signal.

- **Example of spectral leakage?**

– An example of frequency leakage is shown on figure 3.3. where we have sampled 3/2T of signal instead just T.



a) f(t)=sin(ωt), T=8s, SP=1.5s, OP=3/2*T

b) a_0 = 0.1871, a_k = [0.7119 -0.6036 -0.2119]

Figure 3.3. Frequency leaking.

– Instead of getting b_1=1, DFT returns coefficients which at first sight have nothing to do with original signal.
– Red lines on figure 3.3. a) are DFT components with A_k amplitudes.
– Magenta line is result of combining those components.
– DFT tries to combine sine signals whose periods are integer number of 3/2T to aproximate signal whose period is T.

– DFT will think that samples represent single period which we shall call ω_{DFT} and therefore we have:

$$\omega = \frac{2\pi}{T} \tag{3.4}$$

$$T_{DFT} = \frac{3}{2}T \tag{3.5}$$

$$\omega_{DFT} = \frac{2\pi}{T_{DFT}} = \frac{2\pi}{\frac{3}{2}T} = \frac{4\pi}{3T} \tag{3.6}$$

where:
- T – period of original signal,
- ω – frequency of original signal,
- T_{DFT} – DFT thinks that signal period is 3/2T,
- ω_{DFT} – DFT thinks that signal base frequency is 4π/3T.

- **Windowing.**

– Frequency leakage can be reduced by multiplying sampled signal with some window.
– Windowing gently tapers signal to zero at both ends.

- **Hamming window.**

– Hamming window is typically used for this purpose, and it is defined as follows:

$$w(k) = 0.54 - 0.46\cos\left(\frac{2\pi k}{N}\right) \quad , \qquad 1 \le k \le N \tag{3.7}$$

– Using MATLAB command `plot(hamming(100))` we can show that Hamming window, for N=100, looks like shown on figure 3.4:

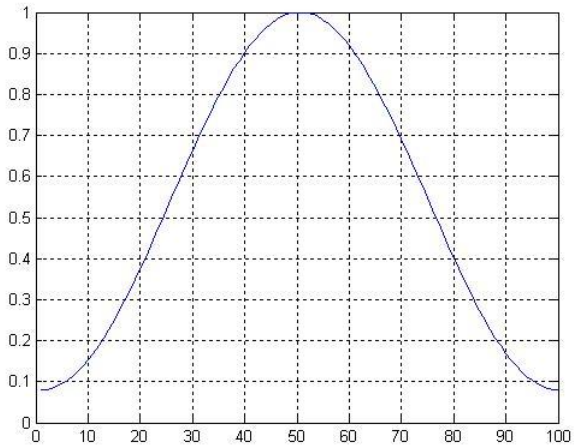


Fig.3.4 Hamming window for N=100.

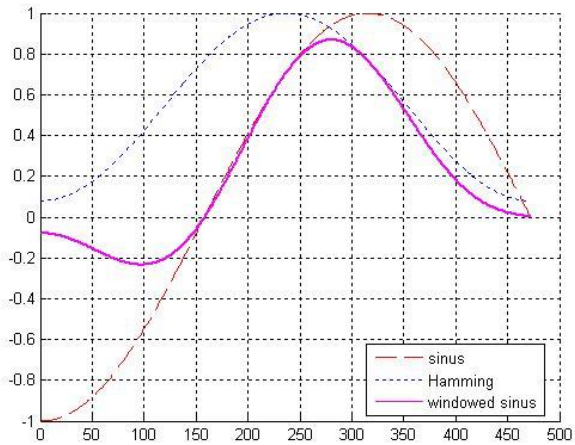


Figure 3.5. Using Hamming window.

– Figure 3.5. shows how Hamming window changes sine signal by reducing discontinuaty at the ends of sampled period.
– Sampled signal is windowed by multiplying it's values with those of Hamming window at the same descrete time steps k:

$$s_w(k) = s_f(k) \cdot w(k) \tag{3.8}$$

where: $s_f(k)$ – value of frame at descrete time step k,
$w(k)$ – value of Hamming window at descrete time step k,
$s_w(k)$ – value of resulting windowed signal at descrete time step k.

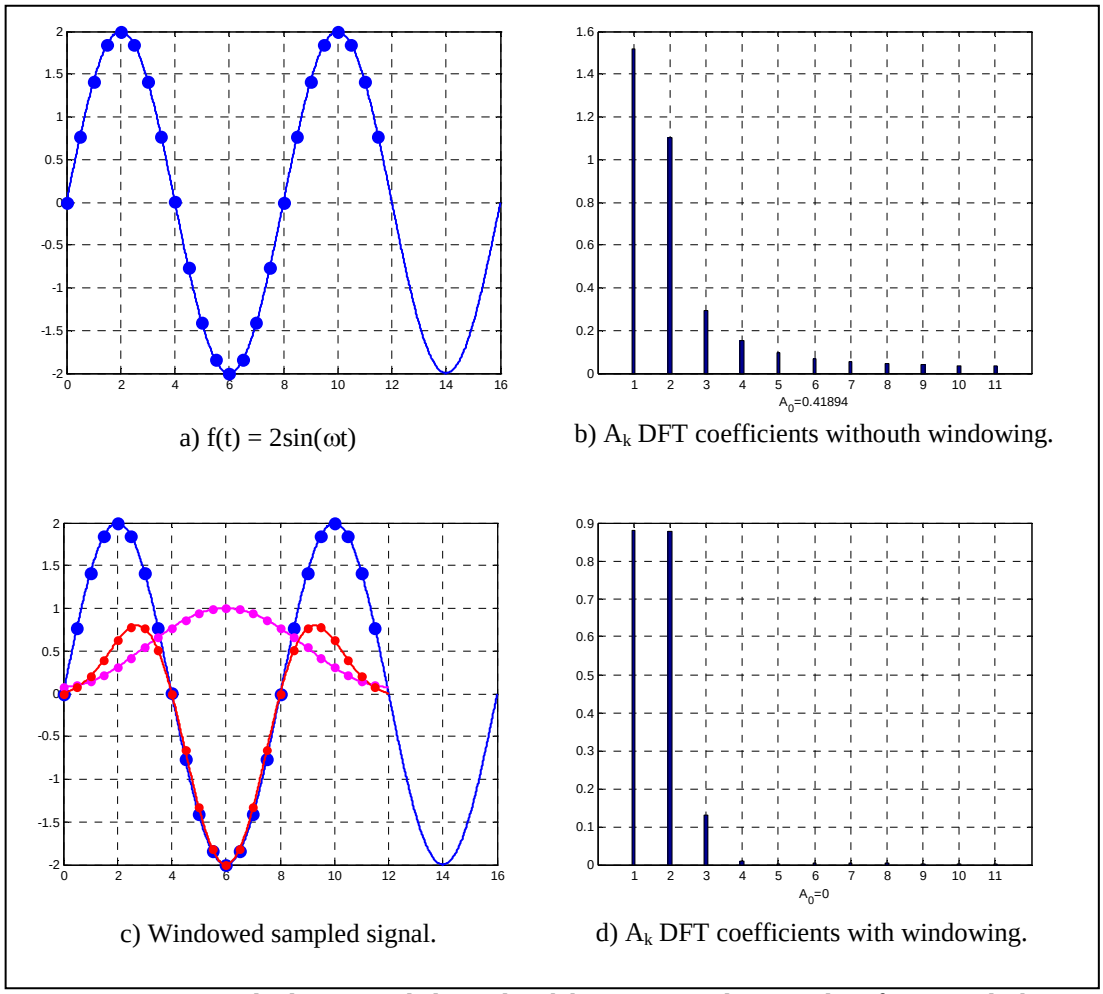
– Figure 3.5. was created using following MATLAB script:

```
s=sin([-pi/2:0.01:pi])';
h=hamming(472);
w=h.*s;

grid on; hold on;
plot(s,'--r'); plot(h,':b'); plot(w,'m','LineWidth',2)
legend('sinus','Hamming window','windowed sinus')
```

- **Uaing Hamming window to detect single frequency.**

– Following example demonstrates how frequency leakage is reduced by multiplying sampled signal with hamming window.



a) f(t) = 2sin(ωt)

b) A_k DFT coefficients withouth windowing.

c) Windowed sampled signal.

d) A_k DFT coefficients with windowing.

Figure 3.6. Multiplying sampled signal with hamming window to reduce frequency leakage.

– Figures Figure 3.6.b) and d) show A_k DFT coefficients with and withouth windowing.
– From Figure 3.6.d) we can calculate real frequency of sin(ωt) as beeing in between frequences which go with A_1 and A_2:

$$\omega = \frac{1}{2}\left(1 \cdot \omega_{DFT} + 2 \cdot \omega_{DFT}\right) = \frac{3}{2}\omega_{DFT} \tag{3.9}$$

– Inserting (3.6) we get:

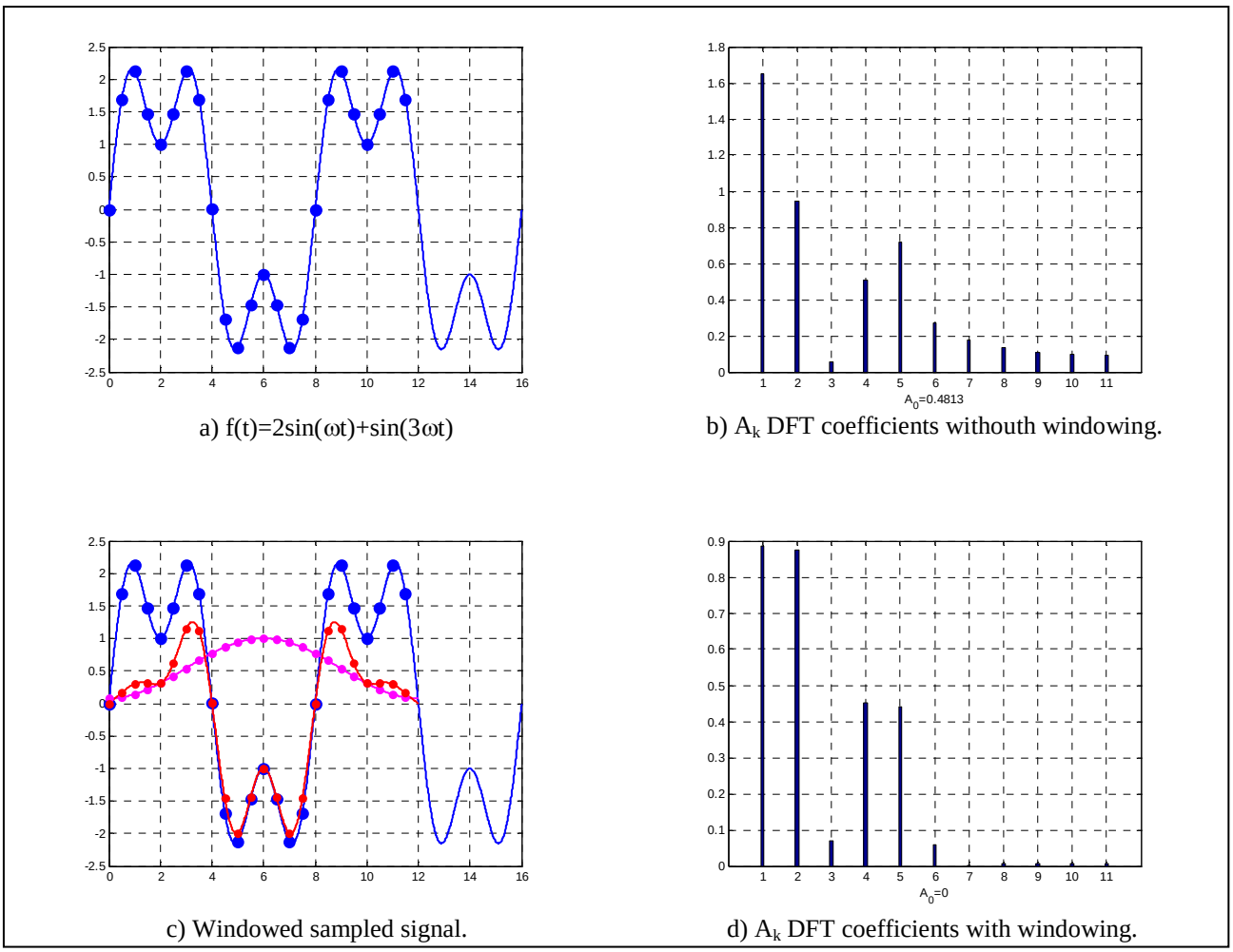
$$\omega = \frac{3}{2}\frac{4\pi}{3T} = \frac{2\pi}{T} \tag{3.10}$$

– We can also estimate signal's amplitude like this:

$$A_1 + A_2 = 0.88 + 0.87 = 1.75$$

- **Using Hamming window to detect two frequencies.**

– Following example demonstrates how frequency leakage is reduced by multiplying sampled signal with hamming window.



a) f(t)=2sin(ωt)+sin(3ωt)

b) A_k DFT coefficients withouth windowing.

c) Windowed sampled signal.

d) A_k DFT coefficients with windowing.

Figure 3.7. Multiplying sampled signal with hamming window reduces frequency leakage.

– From 3.7.d) we can calculate second frequency component of f(t) which is:

$$\omega_3 = 3\omega = 3\frac{2\pi}{T} = \frac{6\pi}{T} \tag{3.11}$$

– We can calculate this frequency from DFT coefficients like this:

$$\omega_3 = \frac{1}{2}\left(4\omega_{DFT} + 5\omega_{DFT}\right) = \frac{9}{2}\omega_{DFT} \tag{3.12}$$

– Inserting (3.6) we get:

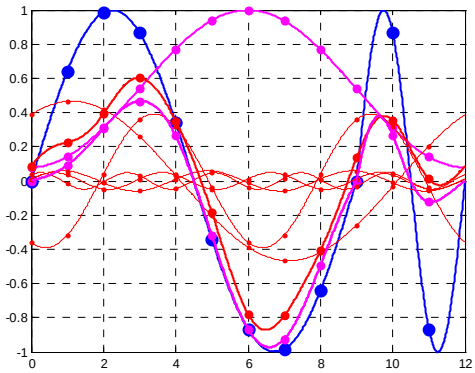
$$\omega = \frac{9}{2}\frac{4\pi}{3T} = \frac{6\pi}{T} \tag{3.13}$$

– We can also estimate amplitude of second component like this:

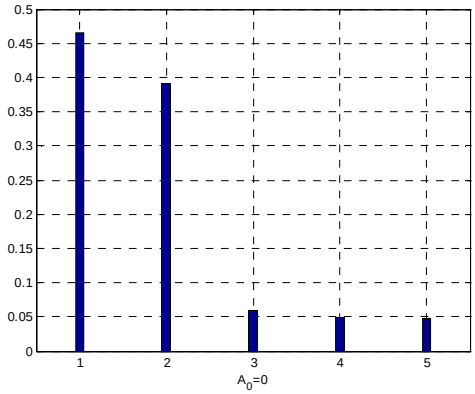
$$A_1 + A_2 = 0.45 + 0.44 = 0.89$$

2.3. Time varying signal

- This chapter shows DFT results when frequency of the signal changes in time but no conslusions are made.
- Figure 3.8. a) shows sine signal whose period changes from 9 to 3 seconds.
- Periods are sampled, windowed and DFT results are given on 3.8. b) which are useless in calculating original frequences.



a) $f(t)=\sin(\omega_1 t)+\sin(\omega_2 t)$



b)

Figure 3.8. DFT of windowed signal with changing frequency.

2.4. Equations

$$a_0 = \frac{1}{N}\sum_{n=1}^{N} f(n) \qquad \text{(signal's mean value)} \qquad (3.14)$$

$$a_k = \frac{2}{N}\sum_{n=1}^{N} f(n)\cos(k\psi n) \quad , \quad k=1,2,\ldots \text{ till } N/2-1 \text{ for even N, or } (N-1)/2 \text{ for odd N} \qquad (3.15)$$

$$b_k = \frac{2}{N}\sum_{n=1}^{N} f(n)\sin(k\psi n) \quad , \quad k=1,2,\ldots \text{ till } N/2-1 \text{ for even N, or } (N-1)/2 \text{ for odd N} \qquad (3.16)$$

3. MATLAB

– This chapter explains how to implement what was explained so far using MATLAB.

3.1. Function ivor_DFS

– This chapter shows how to create custom made MATLAB function ivor_DSF which can calculate DFS coefficients.

– Function must be created in separater MATLAB file like this:

ivor_DFS.m

```
%Calculates coefficients of Descrete Fourier Sequence for descrete signal
%whose values of one period are saved in vector f.
function [a0,a,b] = ivor_DFS(f)

N  = length(f);
w  = 2*pi/N;
t  = (0:N-1);  %Descrete time step. t=0,1,2,...,N-1
a0 = sum(f)/N;

for k = 1:floor(5*N/2)
  a(k) = 2/N*sum(f.*cos(k*w*t));
  b(k) = 2/N*sum(f.*sin(k*w*t));
  if abs(a(k))<0.001 a(k)=0; end  %To avoid displaying small numbers in format 1.0e-013 * 0,0031.
  if abs(b(k))<0.001 b(k)=0; end  %To avoid displaying small numbers in format 1.0e-013 * 0,0031.
end
```

– Here is an example of how this function can be used:

MATLAB Script

```
%NUMBER OF SAMPLES TO TAKE FROM ONE PERIOD.
T = 8;          %Period of signal in seconds.
N = 7;          %Number of samples to take from one period.
w = 2*pi/N;
t = (0:N-1);    %Descrete time steps. t=0,1,2,...,N-1
f = 2+3*cos(3*w*t)-2*sin(2*w*t); %Signal definition.

%CALCULATE DFS.
[a0,ak,bk]=ivor_DFS(f);

%DRAW SIGNALS.
t_cont = (0:0.01:N); f_cont=2+3*cos(3*w*t)-2*sin(2*w*t);   %Continuous signal as reference.
hold on; grid on;
plot(t_cont*T/N, f_cont); xlim([-0.1 T+0.1]);
plot(t*T/N, f,'.','MarkerSize',30);
```

– Here are the results of the above example:

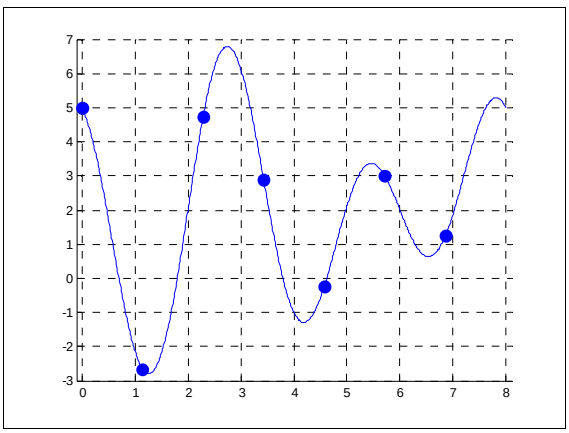


Figure 4.1 $2+3\cos(3\omega t)-2\sin(2\omega t) \Rightarrow a_0=2,\ a_3=3,\ b_2=-2$

3.2. Function DFT_GenerateSignalFrom_a0akbk

– This chapter shows how to create custom made MATLAB function which displays DFT components.
– Function must be created in separater MATLAB file like this:

DFT_GenerateSignalFrom_a0akbk.m

```
%a0,a,b - DFT coefficients: f(t)=a0+a1cos(wt)+b1sin(wt)+...
%OP     - observation period from which samples are taken.
%SP     - sampling period, time in seconds between two samples.
function [f,f_recon] = DFT_GenerateSignalFrom_a0akbk(a0,a,b,OP,SP)
w     = 2*pi/OP;

%CONTINUOUS.
t_sec = (0:SP:OP-SP);    %(0:0.01:OP)-continues, (0:SP:OP-SP)-descrete
f0    = a0*ones(1,length(t_sec));
f_recon=f0;
for k = 1:length(a)
  f(k,:) = a(k)*cos(k*w*t_sec)+b(k)*sin(k*w*t_sec);
  f_recon = f_recon + f(k,:);
  plot(t_sec, f(k,:), 'r.','MarkerSize',20);
end
plot(t_sec, f_recon,'m.','MarkerSize',20);

%DESCRETE.
t_sec = (0:0.01:OP);    %(0:0.01:OP)-continues, (0:SP:OP-SP)-descrete
f_cont= a0*ones(1,length(t_sec));
for k = 1:length(a)
  fk = a(k)*cos(k*w*t_sec)+b(k)*sin(k*w*t_sec);
  f_cont = f_cont + fk;
  plot(t_sec, fk, 'r');
end
plot(t_sec, f_cont,'m','LineWidth',2);

%MARK END OF OP.
line([OP,OP],[min(f_cont),max(f_cont)],'Color','k','LineWidth',2);
```

– Here is an example of how this function can be used:

MATLAB Script

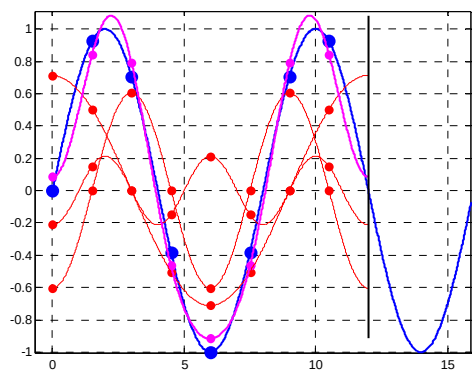
```
%CONTINUOUS SIGNAL.
T      = 8;              %Period of signal in seconds.
w      = 2*pi/T;         %Frequency in radians per second.
t_sec  = (0:0.01:2*T);   %Time in seconds. Show 2 periods of signal.
f_cont = sin(w*t_sec);
figure(1); hold off; plot(t_sec, f_cont,'LineWidth',2); grid;

%DESCRETE SIGNAL.
OP     = 3/2*T;          %Observation period from which samples will be taken.
SP     = 1.5;            %Sampling period. Seconds between two samples.
t_sec  = (0:SP:OP-SP);   %Take samples at this times.
f_desc = sin(w*t_sec);   %Descrete signal containing only samples of continuous signal.
figure(1); hold on; plot(t_sec, f_desc,'.','MarkerSize',30);

%DFS.
[a0,ak,bk]= DFT_Get_a0akbk_FromSamples(f_desc);
[A0,Ak]   = DFT_Get_A0Ak_From_a0akbk(a0,ak,bk);
figure(2); bar(Ak,0.1); xlabel(strcat('A_0=', num2str(a0))); grid;

%SHOW COMPONENTS.
figure(1);
[fk,f_recon] = DFT_GenerateSignalFrom_a0akbk(a0,ak,bk,OP,SP);
```

– Here are the results of the above example:



3.3. Function FFT

- fft is builtin MATLAB function which caclulates C_k of DFT.
- C_0 C_1 C_2
- FFT(a_0) FFT(a_1+jb_1) FFT(a_2+jb_2)

$a_{FFT0}=Na_0$
$a_{FFTk}=N/2a_k$

FFT=2.4142 4.4495 - 0.0000i

a_0 0.2012
a_1 = 0.7416 -0.5690

3.4. Figure 3.2.

– This script shows DFT results when observation period contains integer number of signal periods.

```
%CONTINUOUS SIGNAL.
T      = 8;              %Period of signal in seconds.
w      = 2*pi/T;         %Frequency in radians per second.
time   = (0:0.01:3*T);   %Time in seconds. Show 2 periods of signal.
f_cont = 3*sin(w*time)+sin(3*w*time);
figure(1); hold off; plot(time, f_cont); grid;

%DESCRETE SIGNAL.
OT     = 3*T;            %Observation period from which samples will be taken.
SP     = 1;              %Sampling period. Seconds between two samples.
time   = (0:SP:OT-SP);   %Take samples at this times.
f_desc = 3*sin(w*time)+sin(3*w*time);  %Descrete signal containing only samples of continuous signal.
figure(1); hold on; plot(time, f_desc,'.','MarkerSize',30);

%DFS.
[a0,ak,bk,A]=ivor_DFS(f_desc)
figure(2); bar(A,0.1); xlabel(strcat('A_0=', int2str(a0))); grid;
```

3.5. Figure 3.3.

– This script demonstrates frequency leakage.

```
%CONTINUOUS SIGNAL.
T      = 8;              %Period of signal in seconds.
w      = 2*pi/T;         %Frequency in radians per second.
t_sec  = (0:0.01:2*T);   %Time in seconds. Show 2 periods of signal.
f_cont = sin(w*t_sec);
figure(1); hold off; plot(t_sec, f_cont,'LineWidth',2); grid;

%DESCRETE SIGNAL.
N      = 8;              %Number of samples to take.
SP     = 1.5;            %Sampling period. Seconds between two samples.
OP     = N*SP;           %Observation period from which samples will be taken.
t_sec  = (0:SP:OP-SP);   %Take samples at this times.
f_desc = sin(w*t_sec);   %Descrete signal containing only samples of continuous signal.
figure(1); hold on; plot(t_sec, f_desc,'.','MarkerSize',30);

%DFS.
[a0,ak,bk]= DFT_Get_a0akbk_FromSamples(f_desc);
[A0,Ak]   = DFT_Get_A0Ak_From_a0akbk(a0,ak,bk);
figure(2); bar(Ak,0.1); xlabel(strcat('A_0=', num2str(a0))); grid;

%SHOW COMPONENTS.
figure(1);
[fk,f_recon] = DFT_GenerateSignalFrom_a0akbk(a0,ak,bk,OP,SP);
```

3.6. Figure 3.6.

– This script demonstrates how windowing sampled signal reduces frequency leakage.

```
%CONTINUOUS SIGNAL.
T      = 8;              %Period of signal in seconds.
w      = 2*pi/T;         %Frequency in radians per second.
t_cont = (0:0.01:2*T);   %Time in seconds. Show 2 periods of signal.
f_cont = 2*sin(w*t_cont);
figure(1); hold off; plot(t_cont, f_cont,'LineWidth',2); grid;

%DESCRETE SIGNAL.
N      = 24;             %Number of samples to take.
SP     = 0.5;            %Sampling period. Seconds between two samples.
OP     = N*SP;           %Observation period from which samples will be taken.
t_desc = (0:SP:OP-SP);   %Take samples at this times.
f_desc = 2*sin(w*t_desc);  %Descrete signal containing only samples of continuous signal.
figure(1); hold on; plot(t_desc, f_desc,'.','MarkerSize',30);

%DFS.
[a0,ak,bk]= DFT_Get_a0akbk_FromSamples(f_desc);
[A0,Ak]   = DFT_Get_A0Ak_From_a0akbk(a0,ak,bk);
figure(2); bar(Ak,0.1); xlabel(strcat('A_0=', num2str(a0))); grid;

%%
%HAMMING WINDOW.
figure(1);

%CONTINUOUS.
t_cont_OP = (0:0.01:OP);
f_cont_OP = 2*sin(w*t_cont_OP);
h_cont_OP = hamming(length(f_cont_OP))';
plot(t_cont_OP,h_cont_OP,'m','LineWidth',2)
f_cont_wind=h_cont_OP.*f_cont_OP;
plot(t_cont_OP, f_cont_wind,'r','LineWidth',2);

%DESCRETE.
h_desc=hamming(length(f_desc)+1)';
h_desc=h_desc(1:length(h_desc)-1);
plot(t_desc,h_desc,'.m','MarkerSize',20)
f_desc_wind=h_desc.*f_desc;
plot(t_desc, f_desc_wind,'r.','MarkerSize',20);

%DFS.
[a0,ak,bk]= DFT_Get_a0akbk_FromSamples(f_desc_wind);
[A0,Ak]   = DFT_Get_A0Ak_From_a0akbk(a0,ak,bk);
figure(2); bar(Ak,0.1); xlabel(strcat('A_0=', num2str(a0))); grid;
```

3.7. Figure 3.8.

– This script demonstrates DFT results of windowed signal with changing frequency.

```
%CONTINUOUS SIGNAL 1.
T1      = 9;              %Period of signal in seconds.
w1      = 2*pi/T1;        %Frequency in radians per second.
t_cont1= (0:0.01:T1);  %Time in seconds. Show 2 periods of signal.
f_cont1= sin(w1*t_cont1);
figure(1); hold off; plot(t_cont1, f_cont1, 'b', 'LineWidth',2); grid on;

%CONTINUOUS SIGNAL 2.
T2      = 3;              %Period of signal in seconds.
w2      = 2*pi/T2;        %Frequency in radians per second.
t_cont2= (0.01:0.01:T2);   %Time in seconds. Show 2 periods of signal.
f_cont2= sin(w2*t_cont2);
figure(1); hold on; plot(T1+t_cont2, f_cont2, 'b', 'LineWidth',2);

%CONTINUOUS SIGNAL 1+2.
t_cont = (0:0.01:T1+T2);
f_cont = [f_cont1,f_cont2];

%DESCRETE SIGNAL.
t_desc = t_cont(1:100:length(t_cont)-1);  %Descrete signal containing only samples of continuous signal.
f_desc = f_cont(1:100:length(f_cont)-1);  %Descrete signal containing only samples of continuous signal.
figure(1); hold on; plot(t_desc, f_desc,'.','MarkerSize',30);

%CONTINUOUS HAMMING.
h_cont = hamming(length(t_cont));
plot(t_cont,h_cont,'m','LineWidth',2);

%CONTINUOUS WINDOWED SIGNAL.
f_cont_wind = h_cont'.*f_cont;
plot(t_cont, f_cont_wind,'m','LineWidth',2);

%DESCRETE.
h_desc = hamming(length(f_desc)+1)';
h_desc = h_desc(1:length(h_desc)-1);
plot(t_desc,h_desc,'.m','MarkerSize',20)
f_desc_wind=h_desc.*f_desc;
plot(t_desc, f_desc_wind,'m.','MarkerSize',20);

%DFS.
[a0,ak,bk]= DFT_Get_a0akbk_FromSamples(f_desc_wind);
[A0,Ak]   = DFT_Get_A0Ak_From_a0akbk(a0,ak,bk);
figure(2); bar(Ak,0.1); xlabel(strcat('A_0=', num2str(a0))); grid;

%SHOW COMPONENTS.
figure(1);
OP = 12;
SP = 1;
figure(1);
[fk,f_recon] = DFT_GenerateSignalFrom_a0akbk(a0,ak,bk,OP,SP);
plot(t_cont, f_cont_wind,'m','LineWidth',2);
```

4. References

[1] Fourier Sequence of Descrete Signal (DFS).doc